

How a CDCL SAT Solver works



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Abstract

- Demonstrating how a modern SAT solver works on a small example.
- Target Algorithm
 - Conflict-Driven Clause Learning (CDCL)
 - 1-UIP learning + far-backtracking
 - details are omitted (decision heuristics, efficient data structure, etc.)

An Example Problem

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Decide

Implication Graph

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Decide

Implication Graph

x1

x1@1

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Decide

$x1$
|
 $x2$

Implication Graph

$x1@1$

$x2@2$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Decide

x1
|
x2
|
x3

Implication Graph

x3@3

x1@1

x2@2

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Decide

x1
|
x2
|
x3
|
x4

Implication Graph

x3@3

x1@1

x4@4

x2@2

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph

x3@3

x1@1

x4@4

x2@2

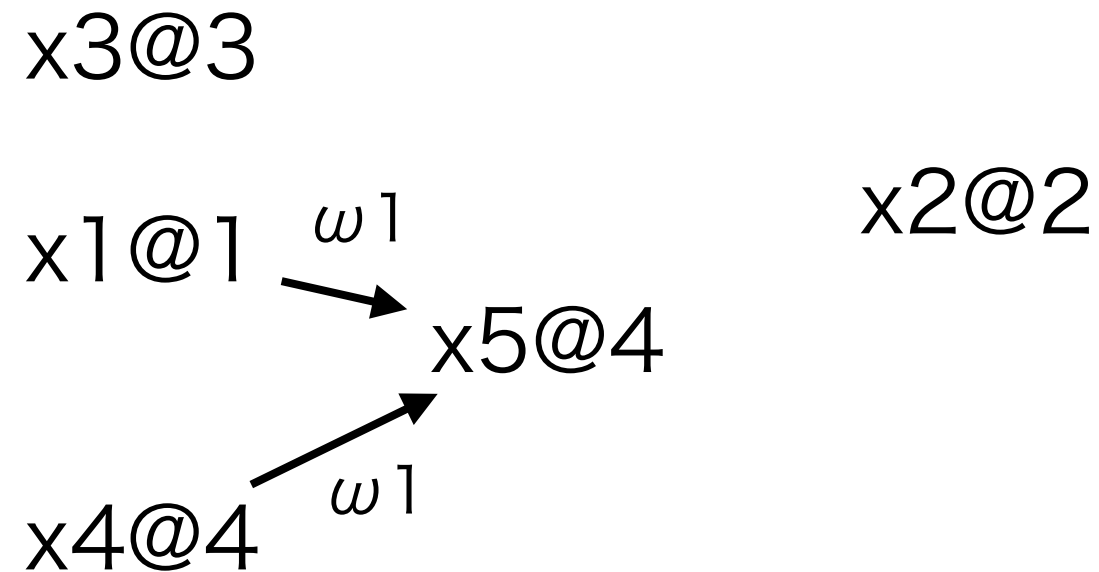
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- $\omega 7: \neg x8 \vee x9$

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



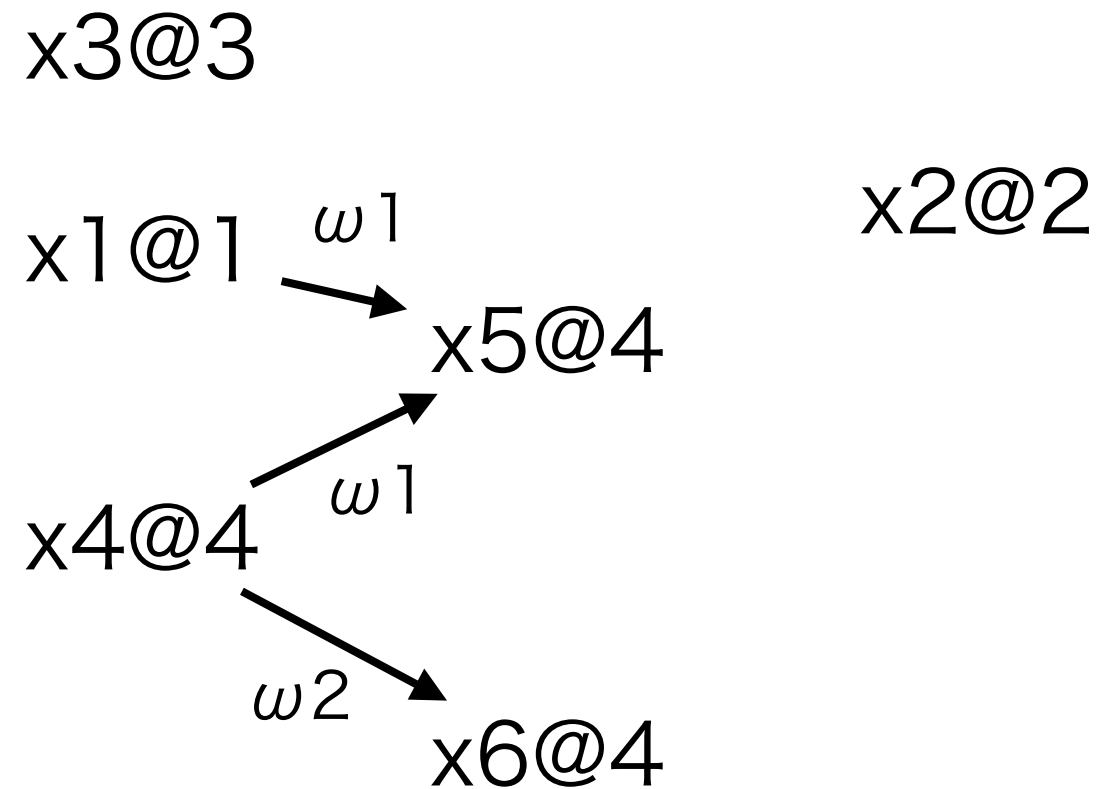
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- $\omega 7: \neg x8 \vee x9$

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



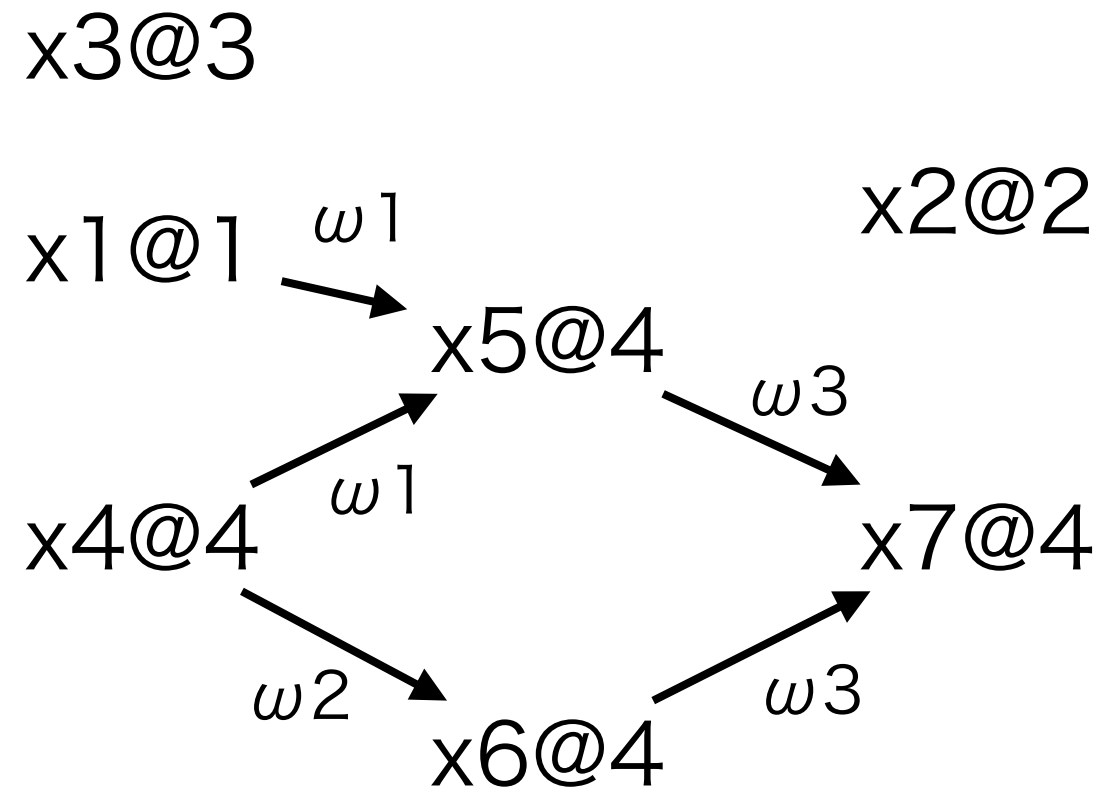
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Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



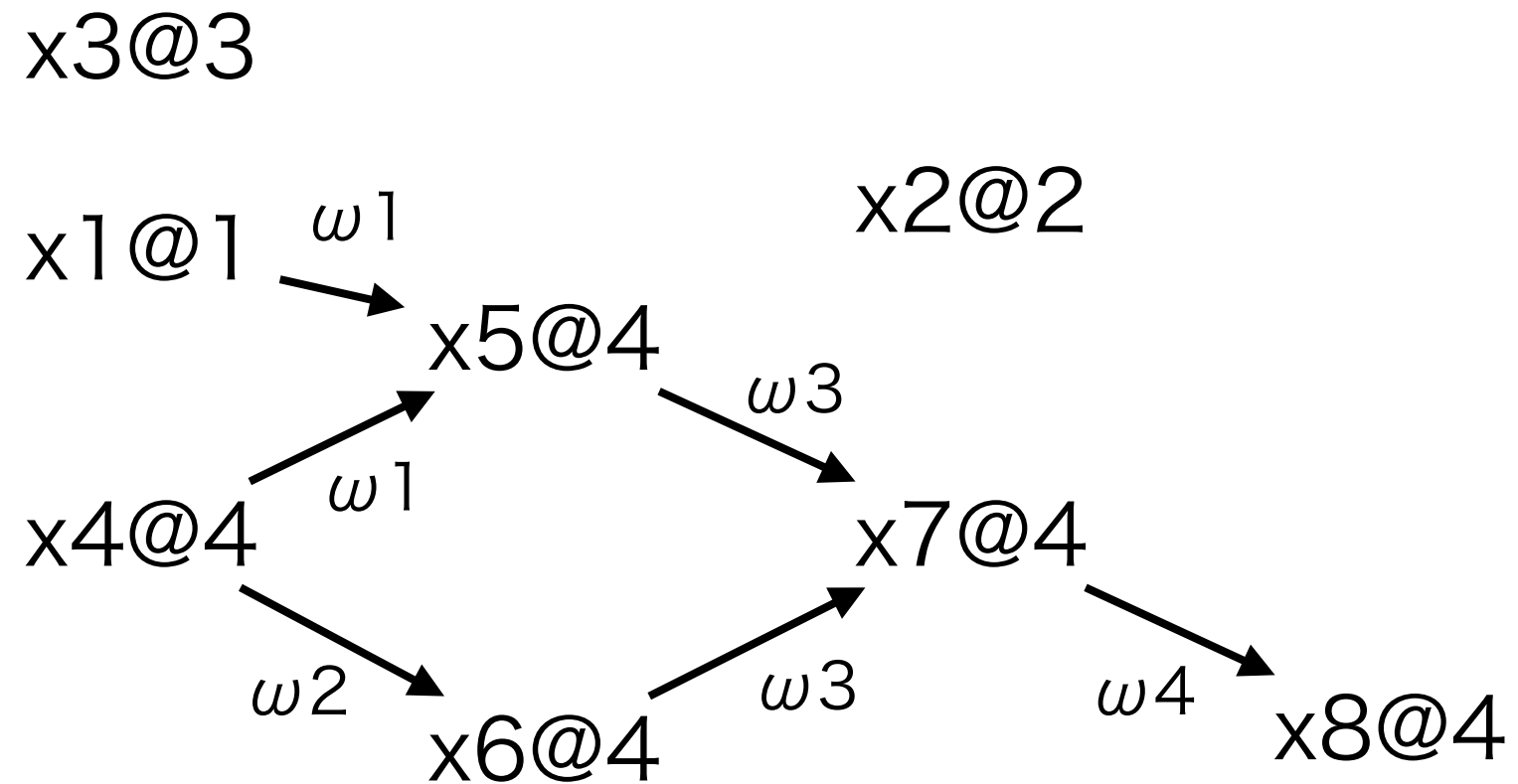
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Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



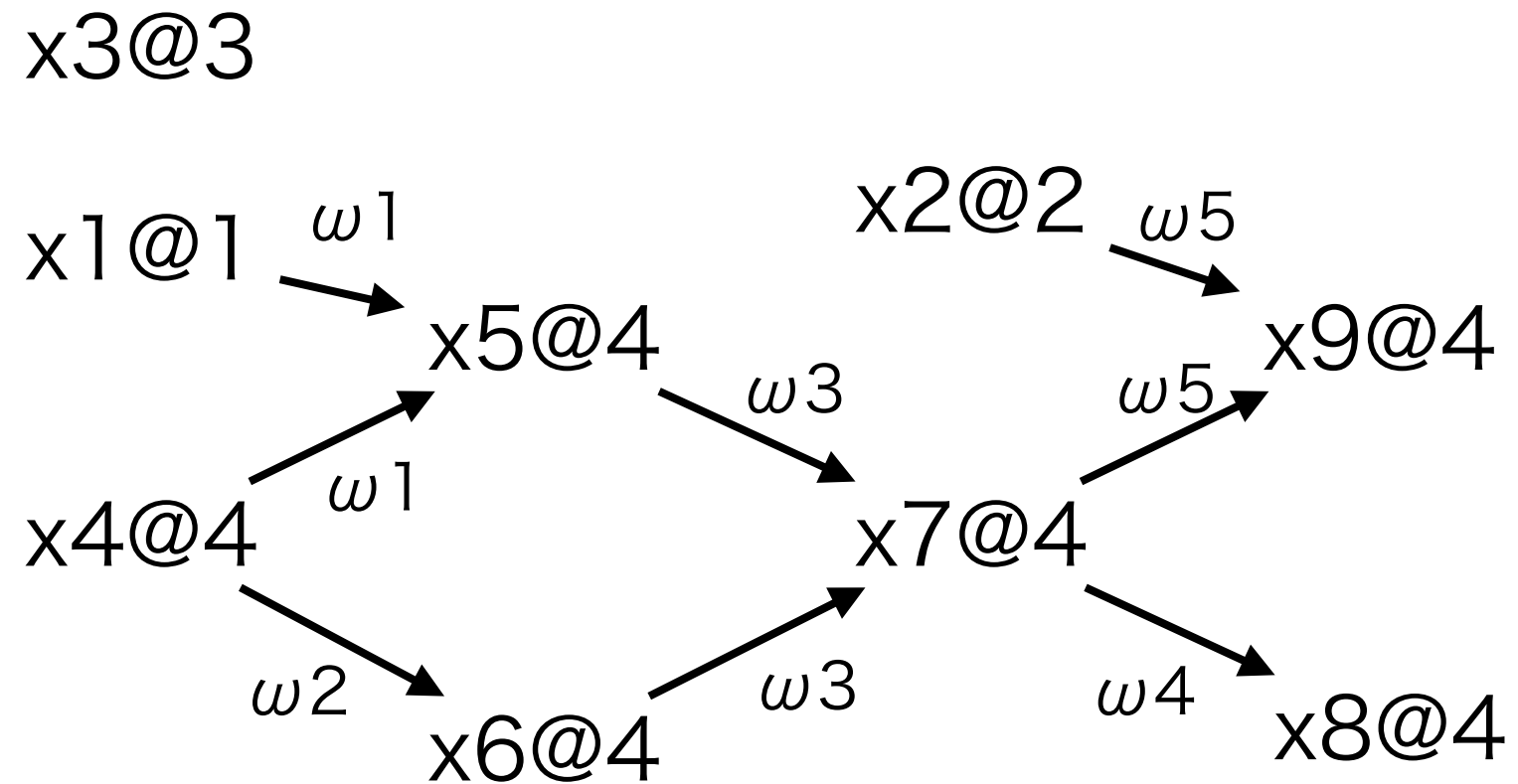
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- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



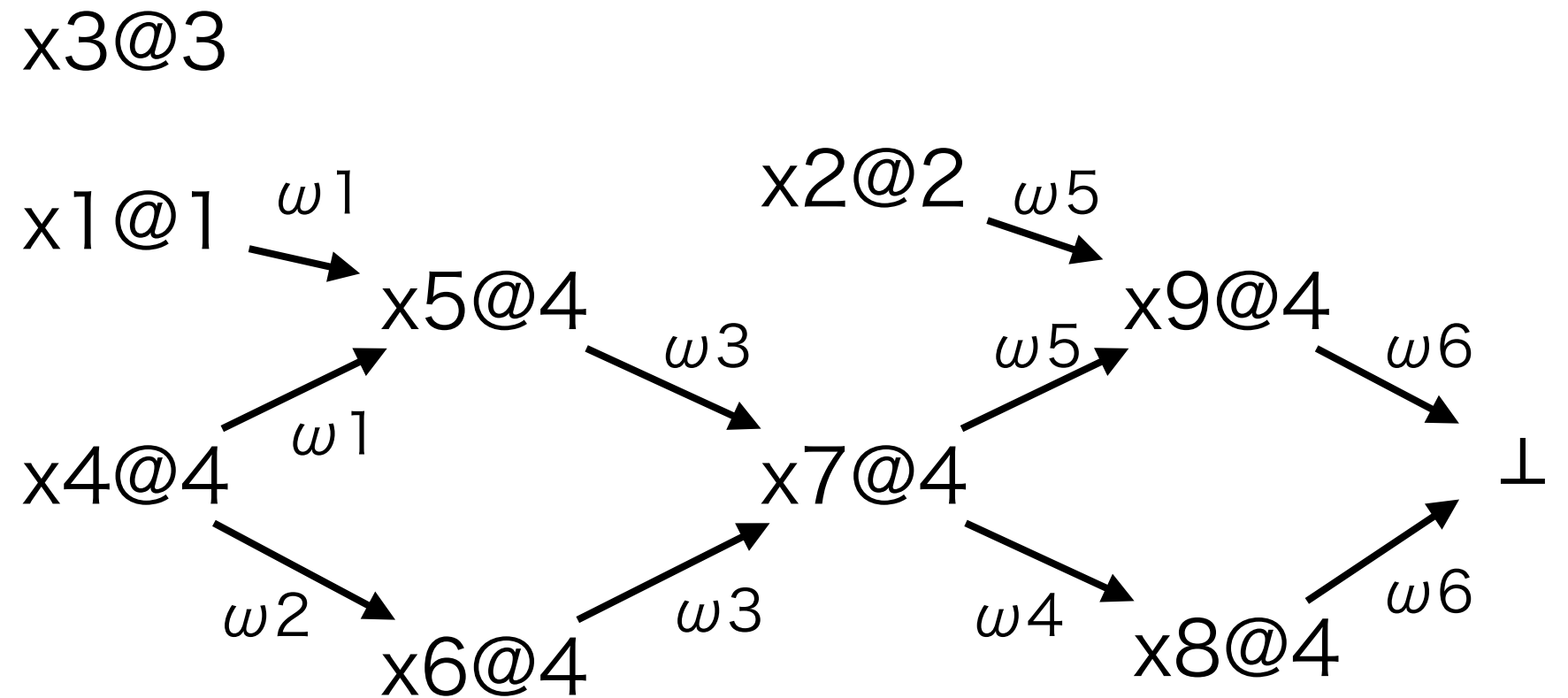
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- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



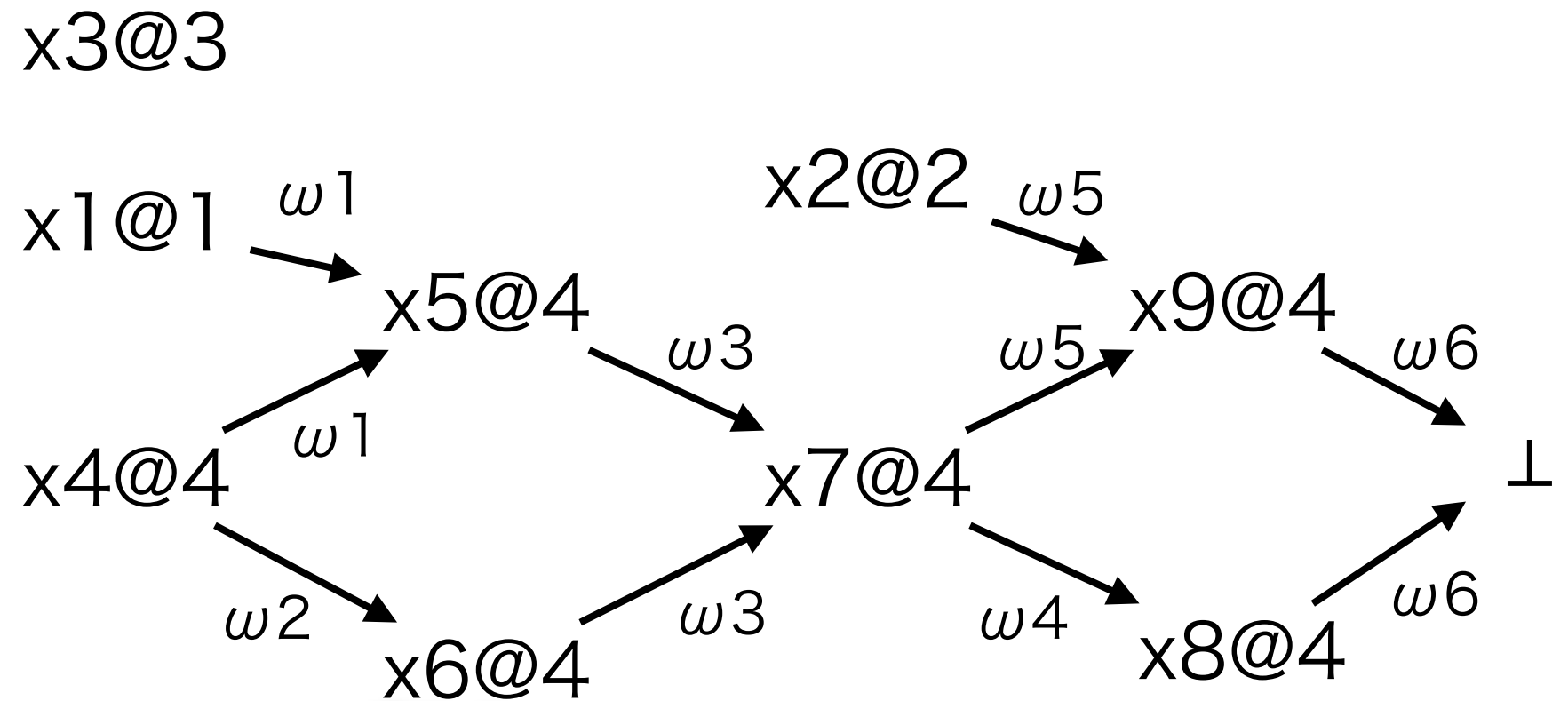
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- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



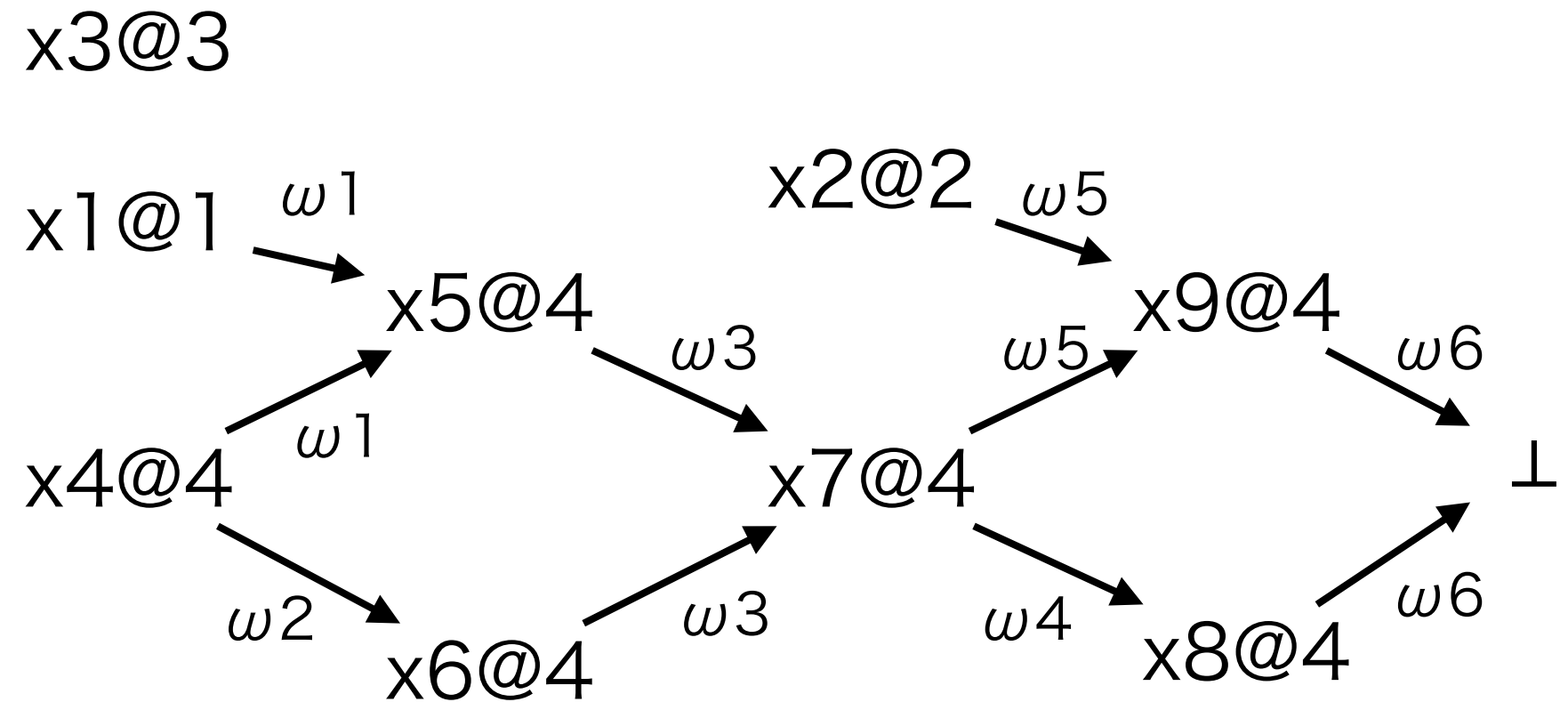
Clause DB

Conflict

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Diagnose

Implication Graph



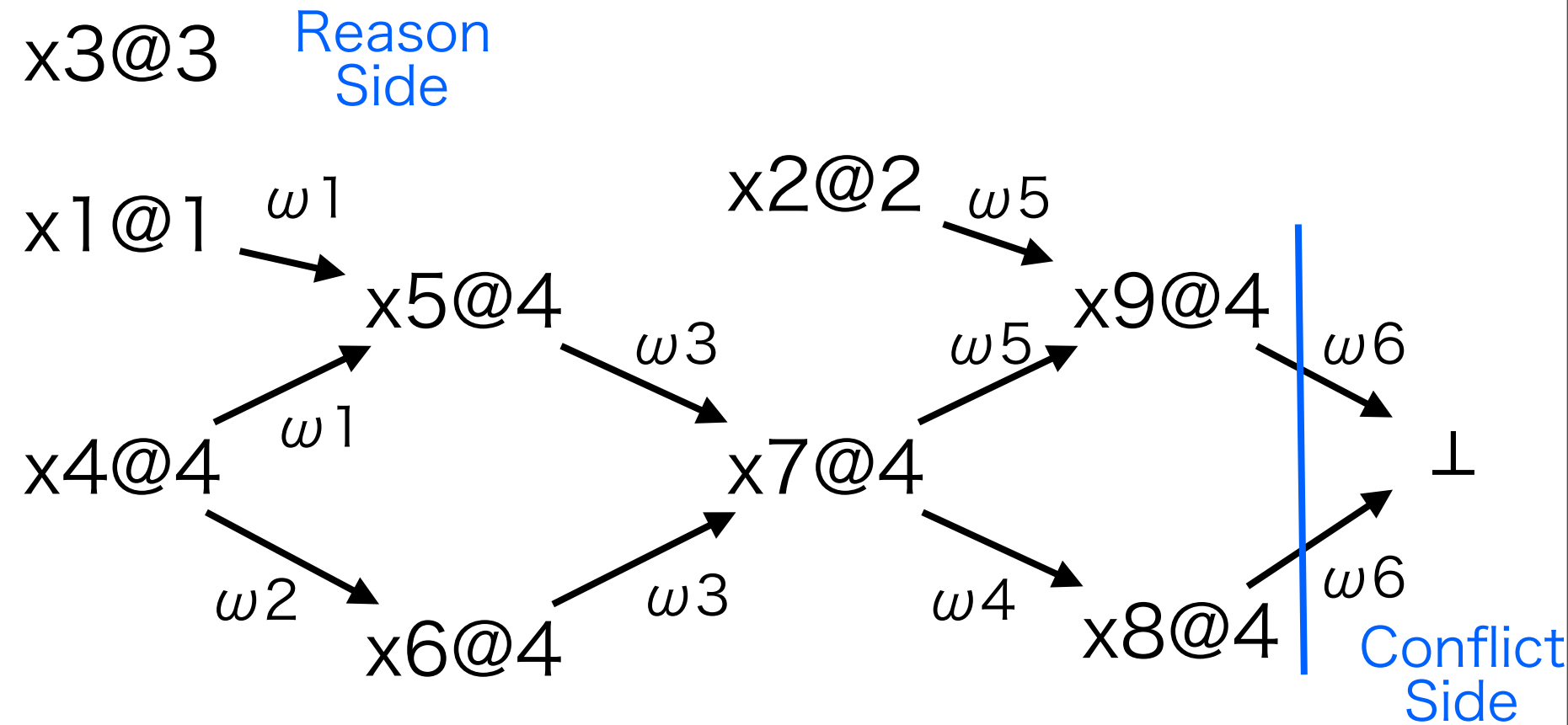
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Diagnose

$\omega 6:$
 $\neg x8 \vee \neg x9$

Implication Graph



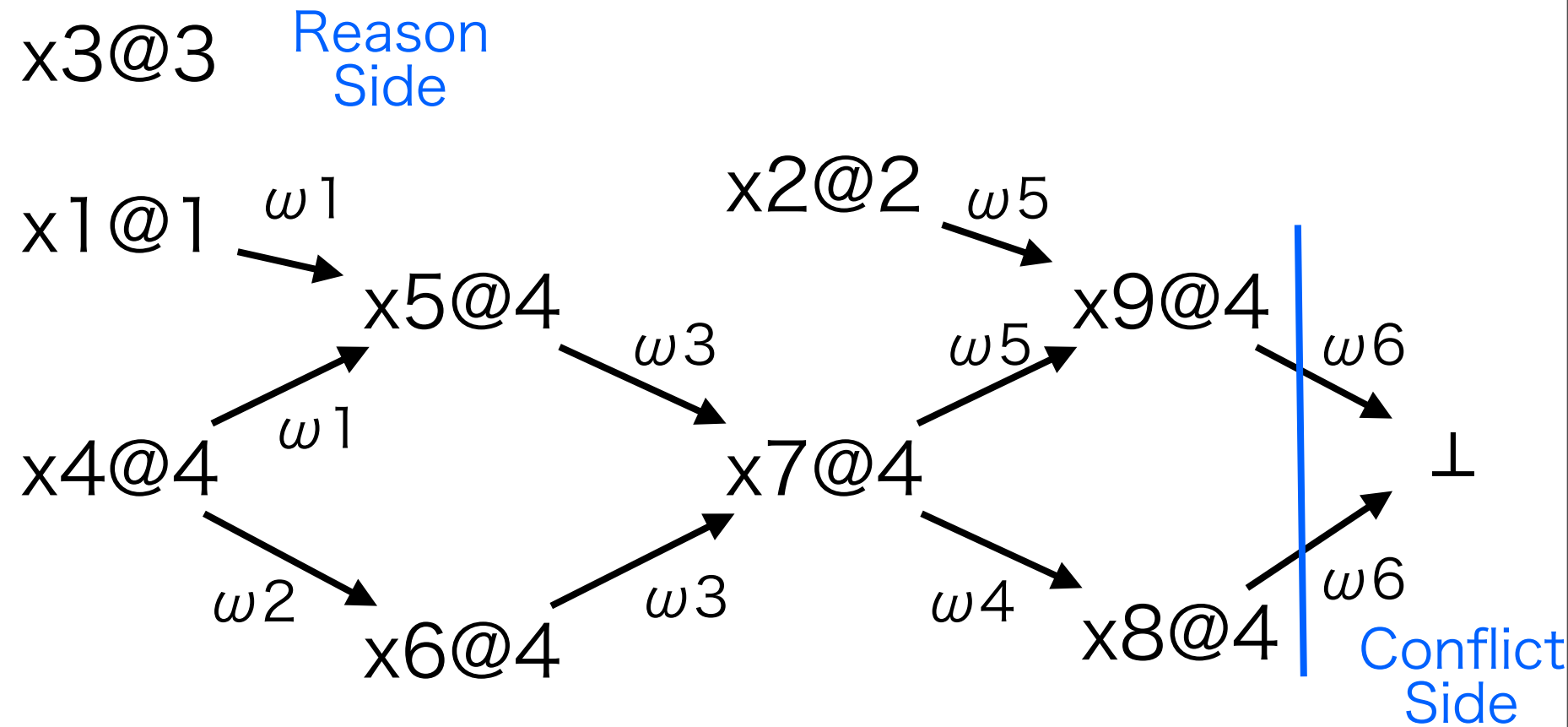
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
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- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Diagnose

$\omega 6: \neg x8 \vee \neg x9$ $\omega 4: \neg x7 \vee x8$

Implication Graph



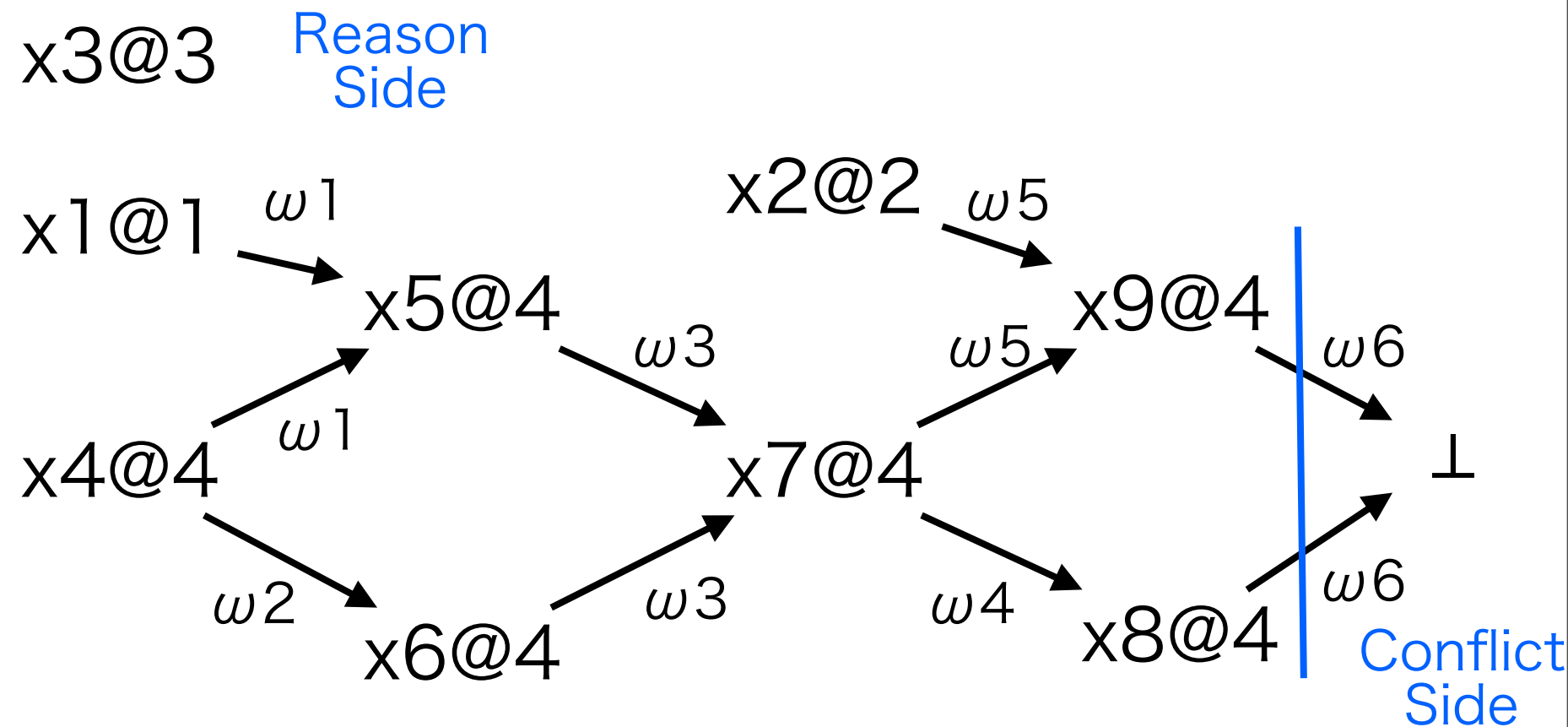
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- $\omega 2: \neg x4 \vee x6$
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- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
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Diagnose

$\omega 6: \neg x8 \vee \neg x9$ $\omega 4: \neg x7 \vee x8$

Implication Graph



Clause DB

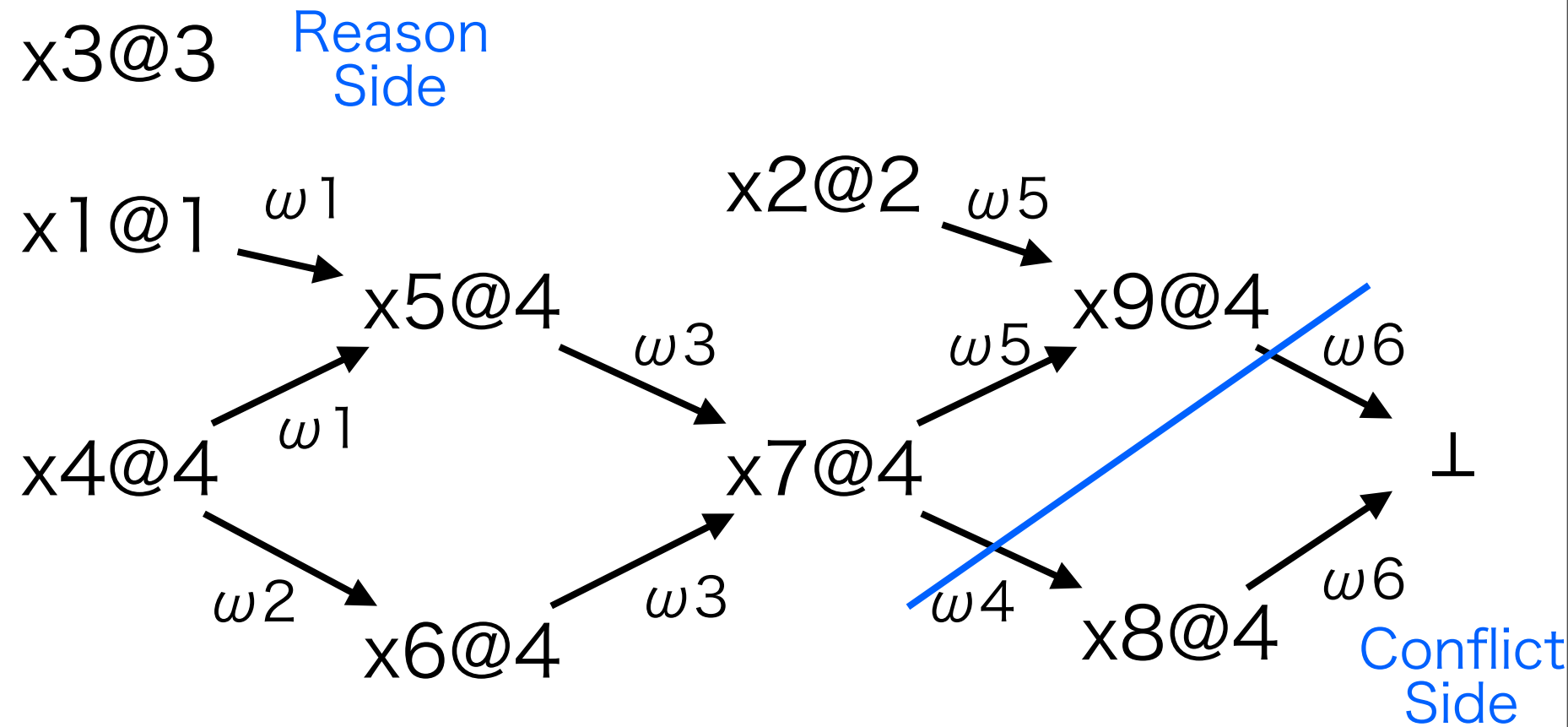
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- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Diagnose

$\omega 6: \neg x8 \vee \neg x9$ $\omega 4: \neg x7 \vee x8$

 $\neg x7 \vee \neg x9$

Implication Graph



Clause DB

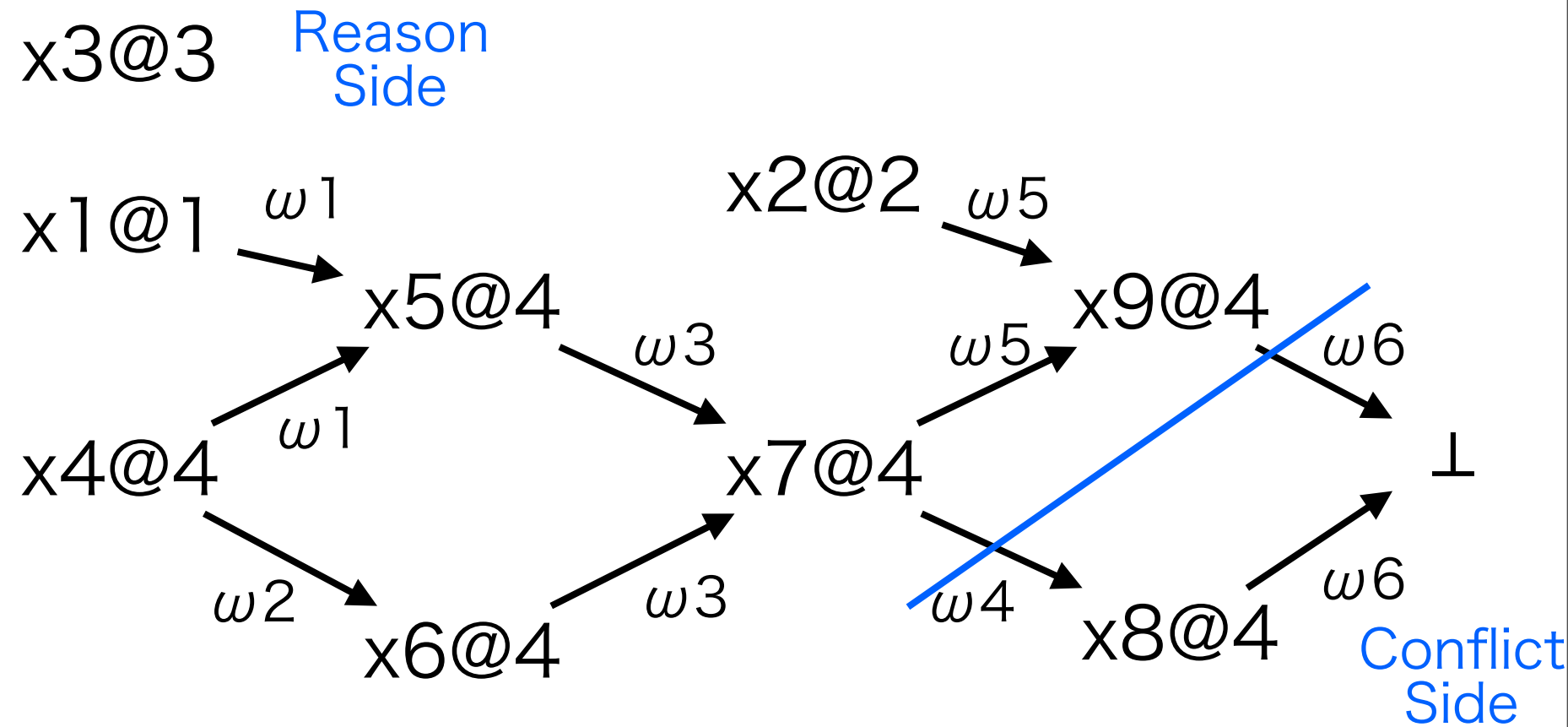
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- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Diagnose

$\omega 6: \neg x8 \vee \neg x9$ $\omega 4: \neg x7 \vee x8$

 $\neg x7 \vee \neg x9$ $\omega 5: \neg x2 \vee \neg x7 \vee x9$

Implication Graph



Clause DB

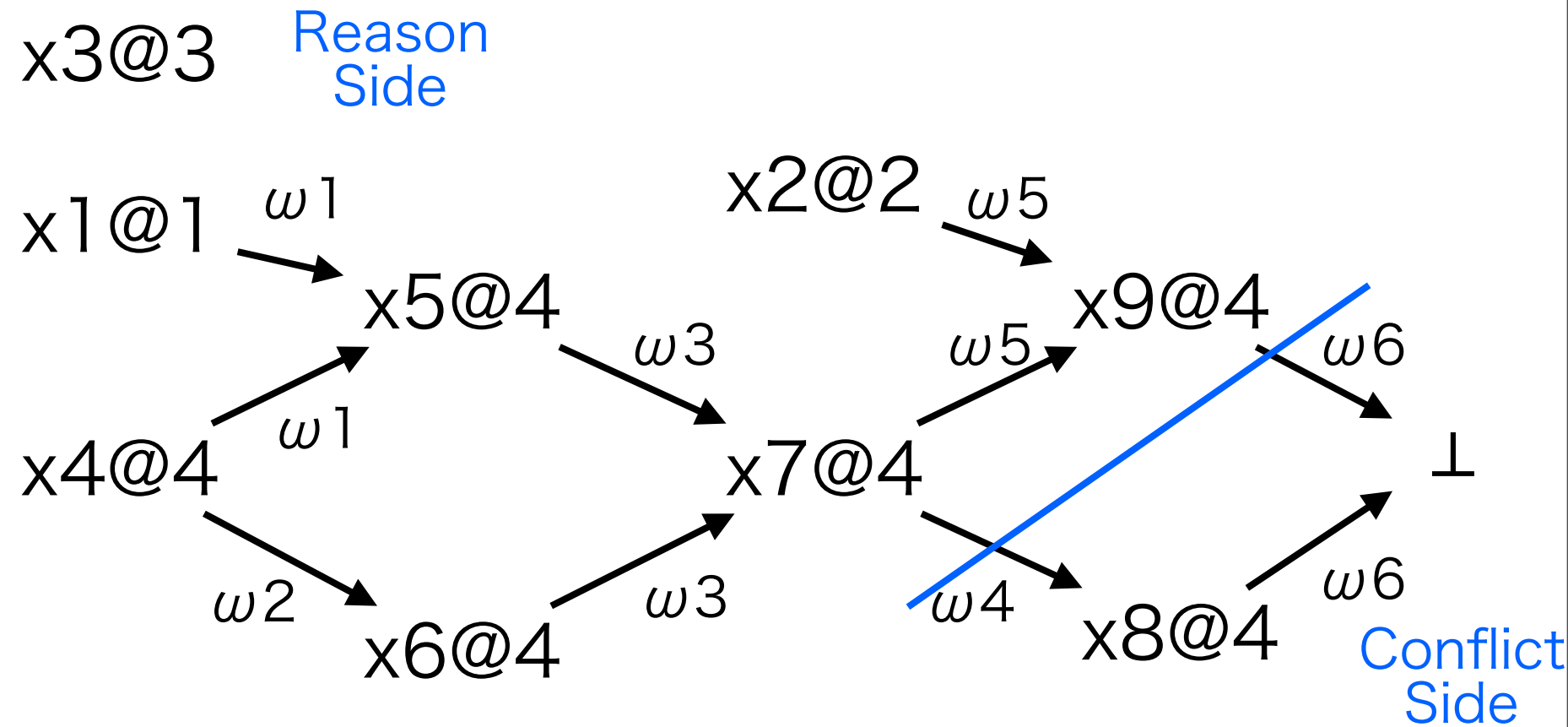
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- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Diagnose

$\omega 6: \neg x8 \vee \neg x9$ $\omega 4: \neg x7 \vee x8$

 $\neg x7 \vee \neg x9$ $\omega 5: \neg x2 \vee \neg x7 \vee x9$

Implication Graph



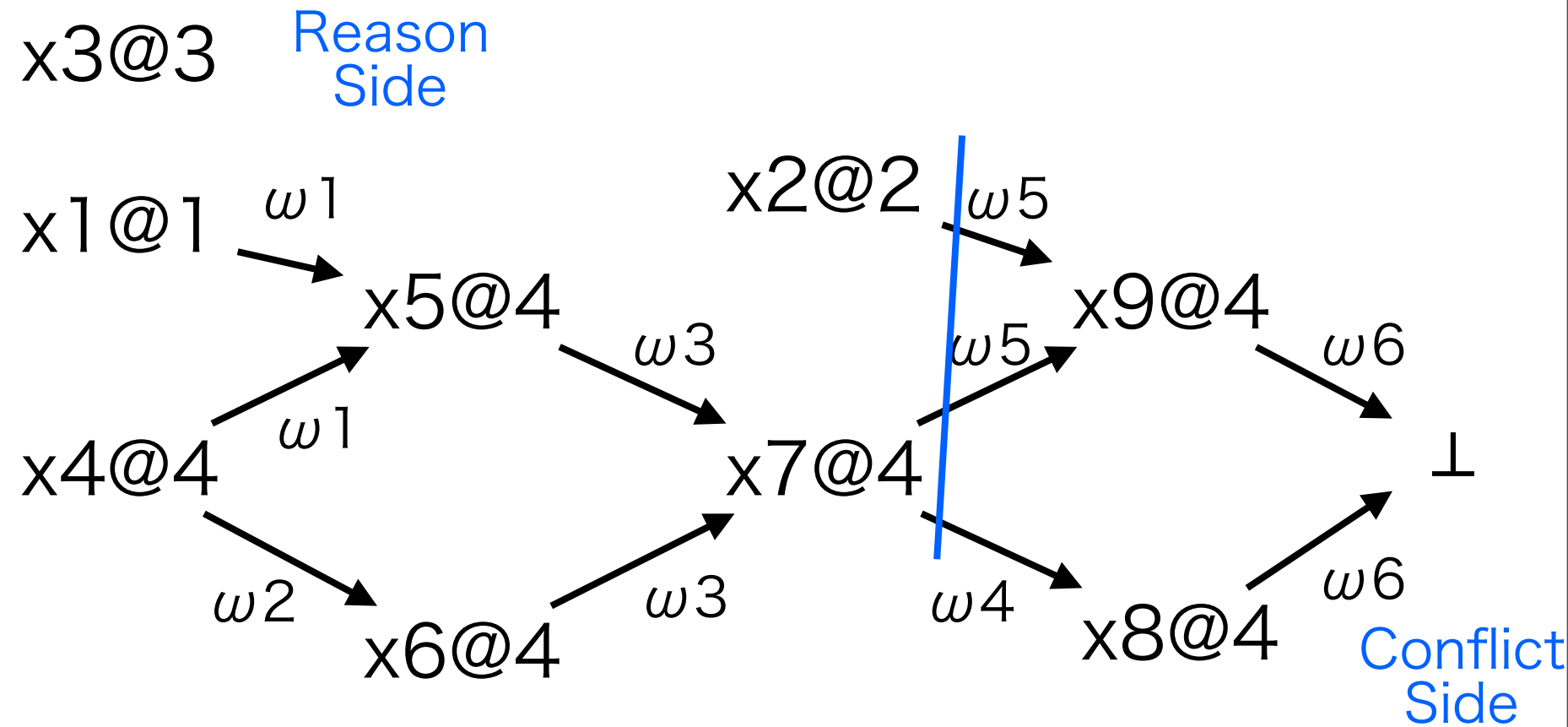
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

Diagnose

$$\begin{array}{c}
 \omega 6: \neg x 8 \vee \neg x 9 \quad \omega 4: \neg x 7 \vee x 8 \\
 \hline
 \neg x 7 \vee \neg x 9 \quad \omega 5: \neg x 2 \vee \neg x 7 \vee x 9 \\
 \hline
 \neg x 2 \vee \neg x 7
 \end{array}$$

Implication Graph



Clause DB

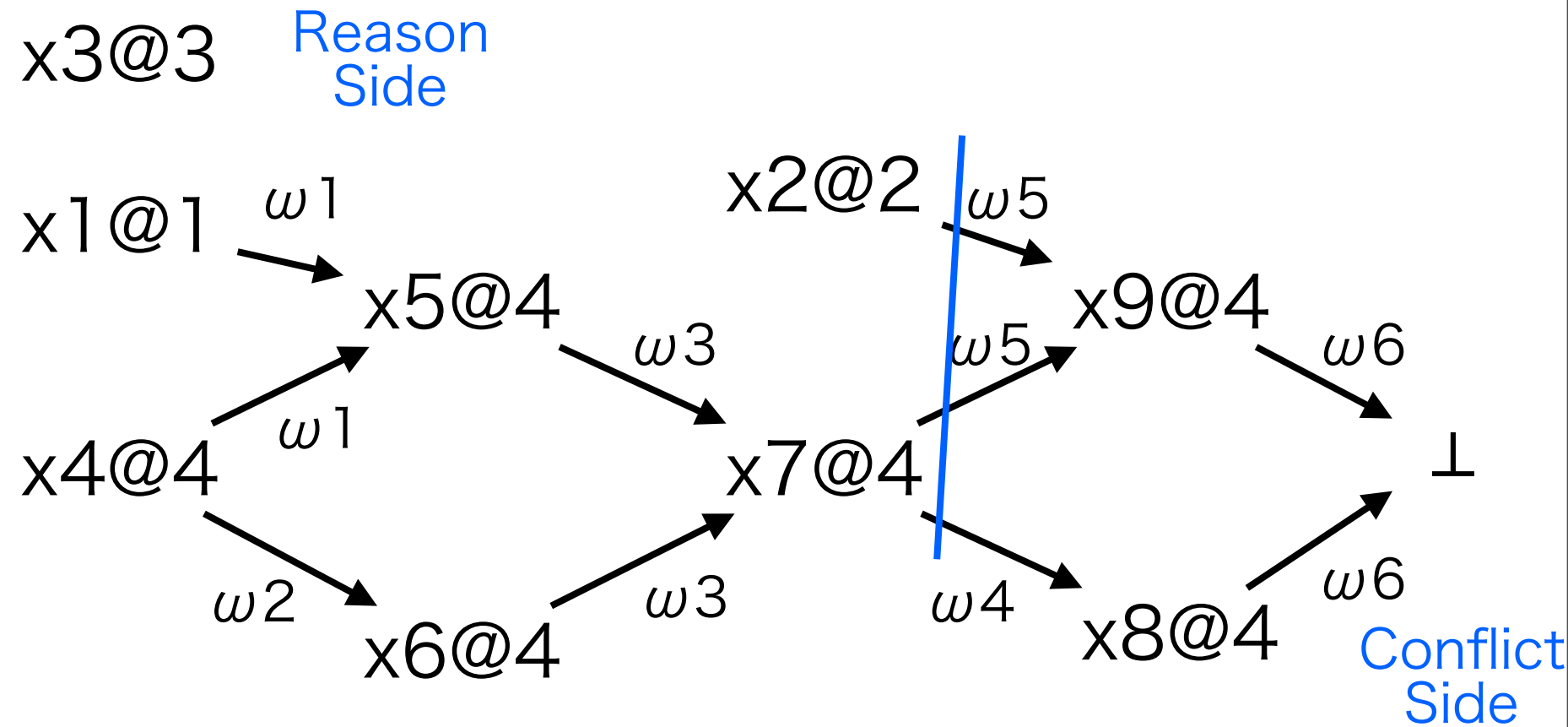
- $\omega 1: \neg x 1 \vee \neg x 4 \vee x 5$
- $\omega 2: \neg x 4 \vee x 6$
- $\omega 3: \neg x 5 \vee \neg x 6 \vee x 7$
- $\omega 4: \neg x 7 \vee x 8$
- $\omega 5: \neg x 2 \vee \neg x 7 \vee x 9$
- $\omega 6: \neg x 8 \vee \neg x 9$
- $\omega 7: \neg x 8 \vee x 9$

Diagnose

$$\begin{array}{c} \omega 6: \neg x 8 \vee \neg x 9 \quad \omega 4: \neg x 7 \vee x 8 \\ \hline \neg x 7 \vee \neg x 9 \quad \omega 5: \neg x 2 \vee \neg x 7 \vee x 9 \\ \hline \neg x 2 \vee \neg x 7 \end{array}$$

It contains one literal
in the current
decision level (i.e. 4).

Implication Graph



Clause DB

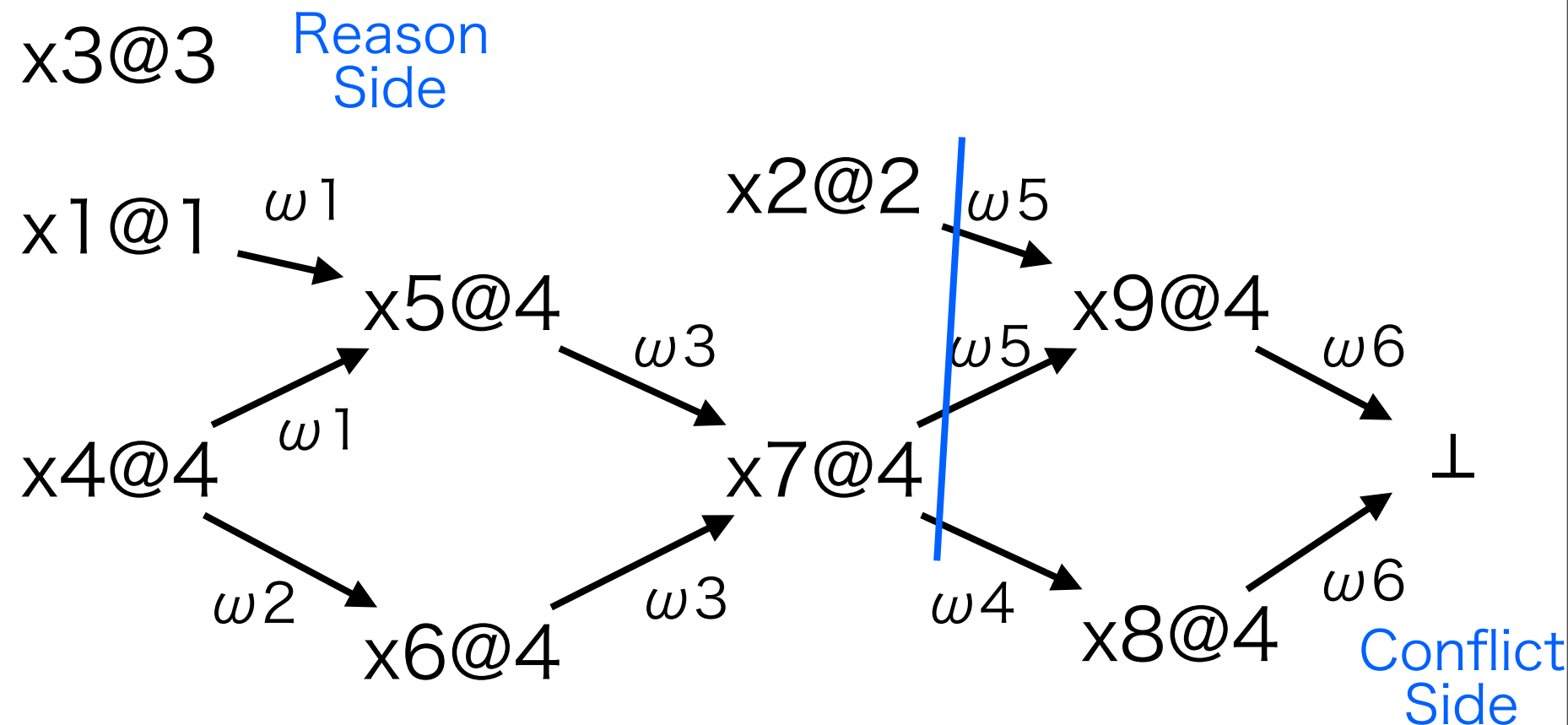
- $\omega 1: \neg x 1 \vee \neg x 4 \vee x 5$
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- $\omega 3: \neg x 5 \vee \neg x 6 \vee x 7$
- $\omega 4: \neg x 7 \vee x 8$
- $\omega 5: \neg x 2 \vee \neg x 7 \vee x 9$
- $\omega 6: \neg x 8 \vee \neg x 9$
- $\omega 7: \neg x 8 \vee x 9$

Diagnose

$$\begin{array}{l} \omega 6: \neg x 8 \vee \neg x 9 \quad \omega 4: \neg x 7 \vee x 8 \\ \hline \neg x 7 \vee \neg x 9 \quad \omega 5: \neg x 2 \vee \neg x 7 \vee x 9 \\ \hline \neg x 2 \vee \neg x 7 \end{array}$$

It contains one literal
in the current
decision level (i.e. 4).

Implication Graph



Clause DB

- $\omega 1: \neg x 1 \vee \neg x 4 \vee x 5$
- $\omega 2: \neg x 4 \vee x 6$
- $\omega 3: \neg x 5 \vee \neg x 6 \vee x 7$
- $\omega 4: \neg x 7 \vee x 8$
- $\omega 5: \neg x 2 \vee \neg x 7 \vee x 9$
- $\omega 6: \neg x 8 \vee \neg x 9$
- $\omega 7: \neg x 8 \vee x 9$
- $\omega 8: \neg x 2 \vee \neg x 7$

New clause is learnt !

Backtrack

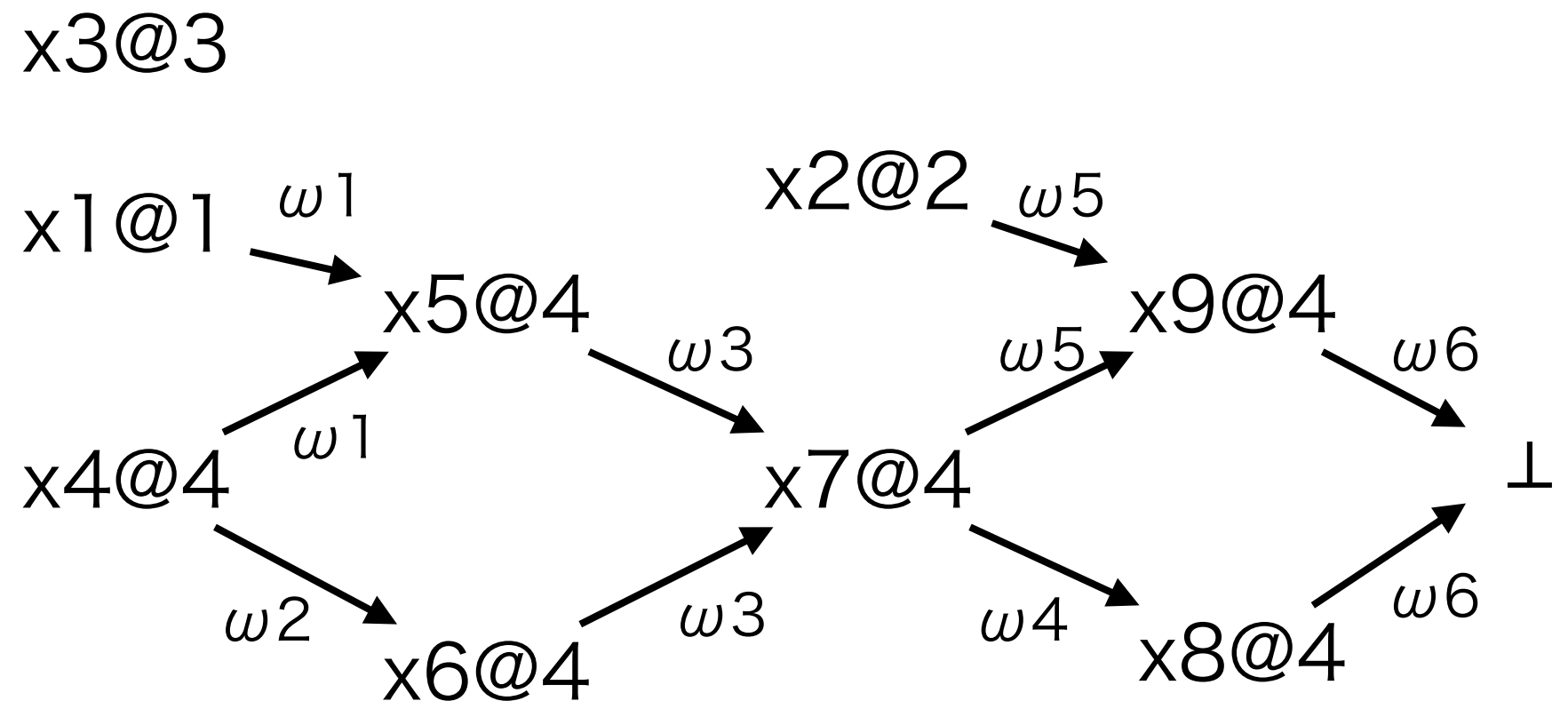
Learnt clause:
 $\neg x_2 \vee \neg x_7$

x_2 was set at level 2
 x_7 was set at level 4

Assertion level of this clause (which is the second highest level) is 2.

$\Rightarrow$ Backtrack to level 2!

Implication Graph



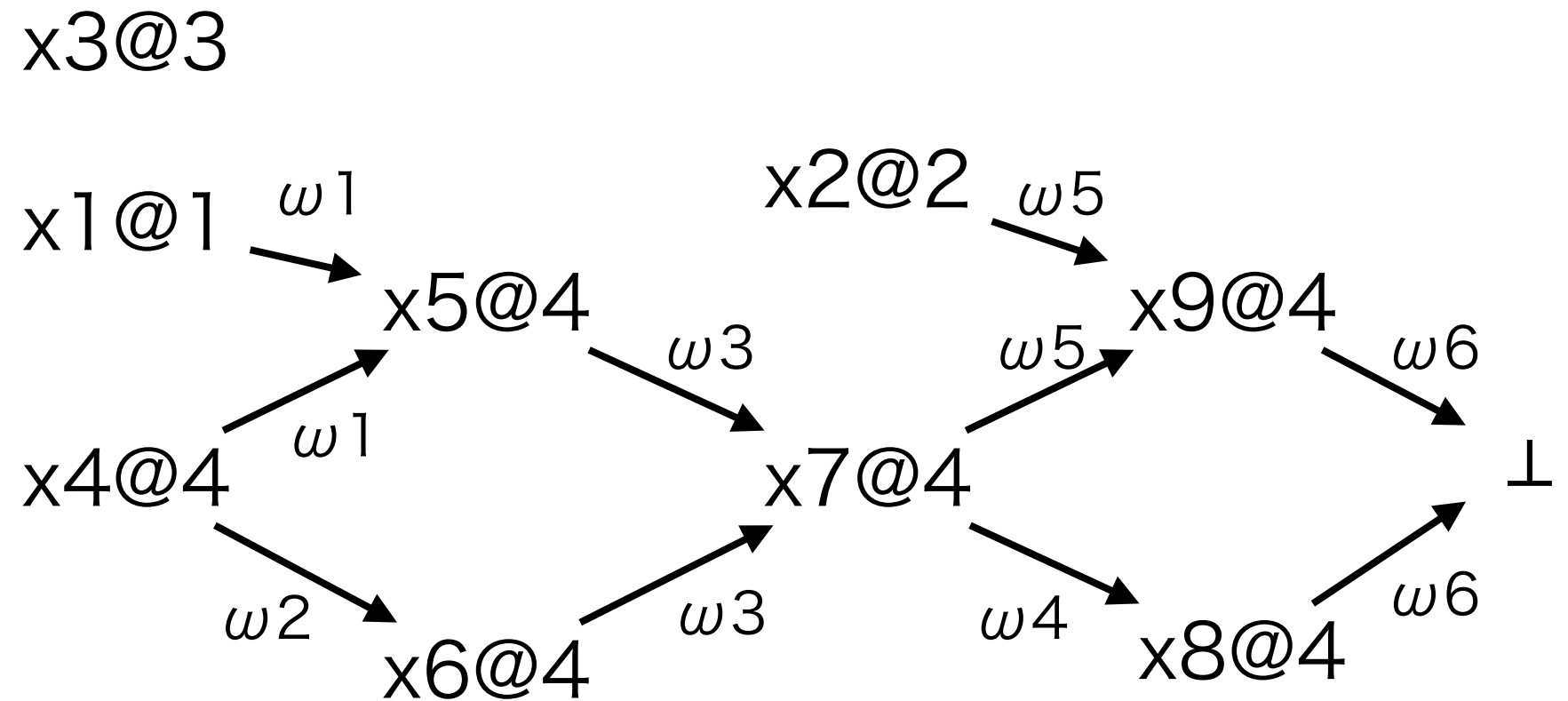
Clause DB

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- $\omega_2: \neg x_4 \vee x_6$
- $\omega_3: \neg x_5 \vee \neg x_6 \vee x_7$
- $\omega_4: \neg x_7 \vee x_8$
- $\omega_5: \neg x_2 \vee \neg x_7 \vee x_9$
- $\omega_6: \neg x_8 \vee \neg x_9$
- $\omega_7: \neg x_8 \vee x_9$
- $\omega_8: \neg x_2 \vee \neg x_7$

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



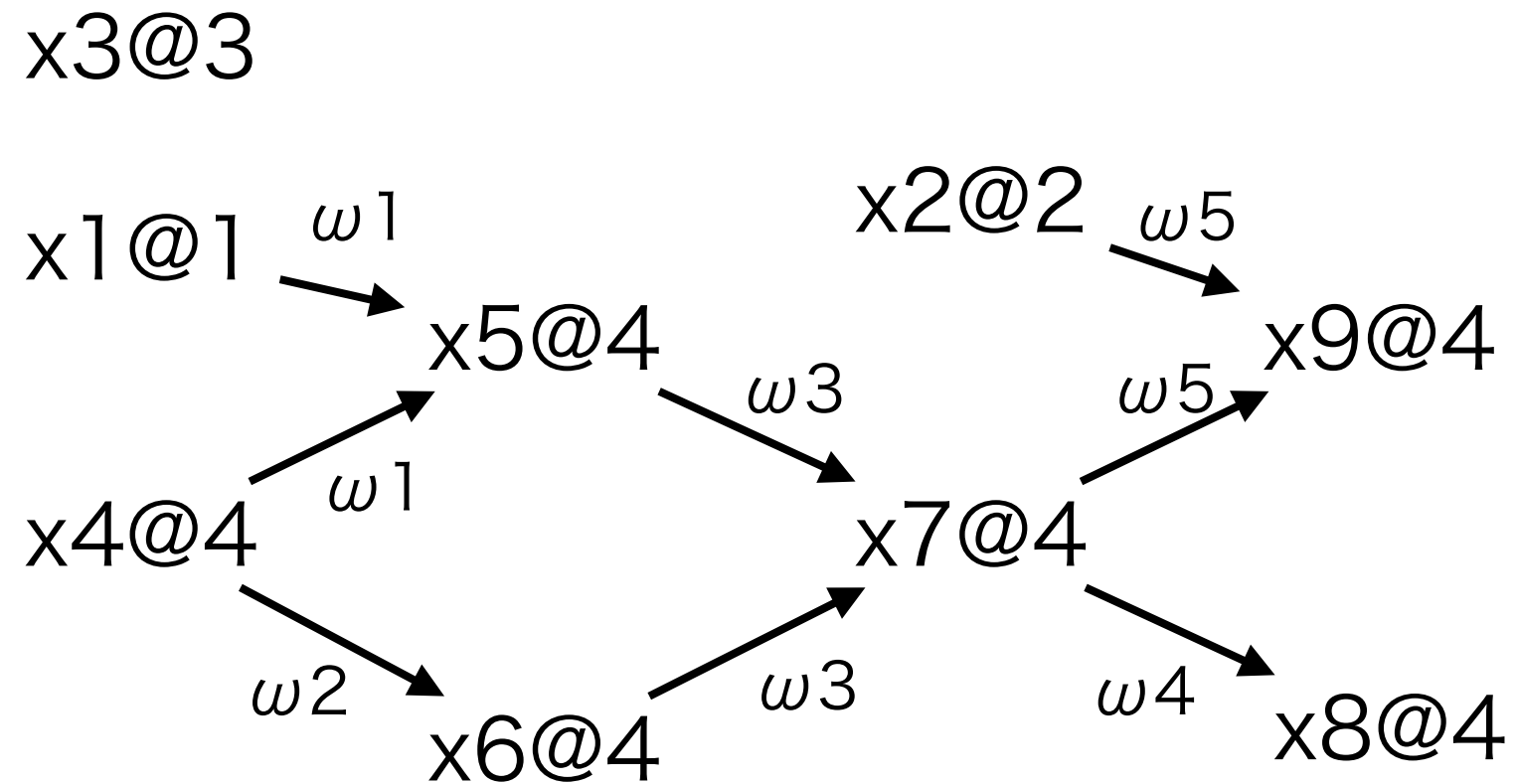
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- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



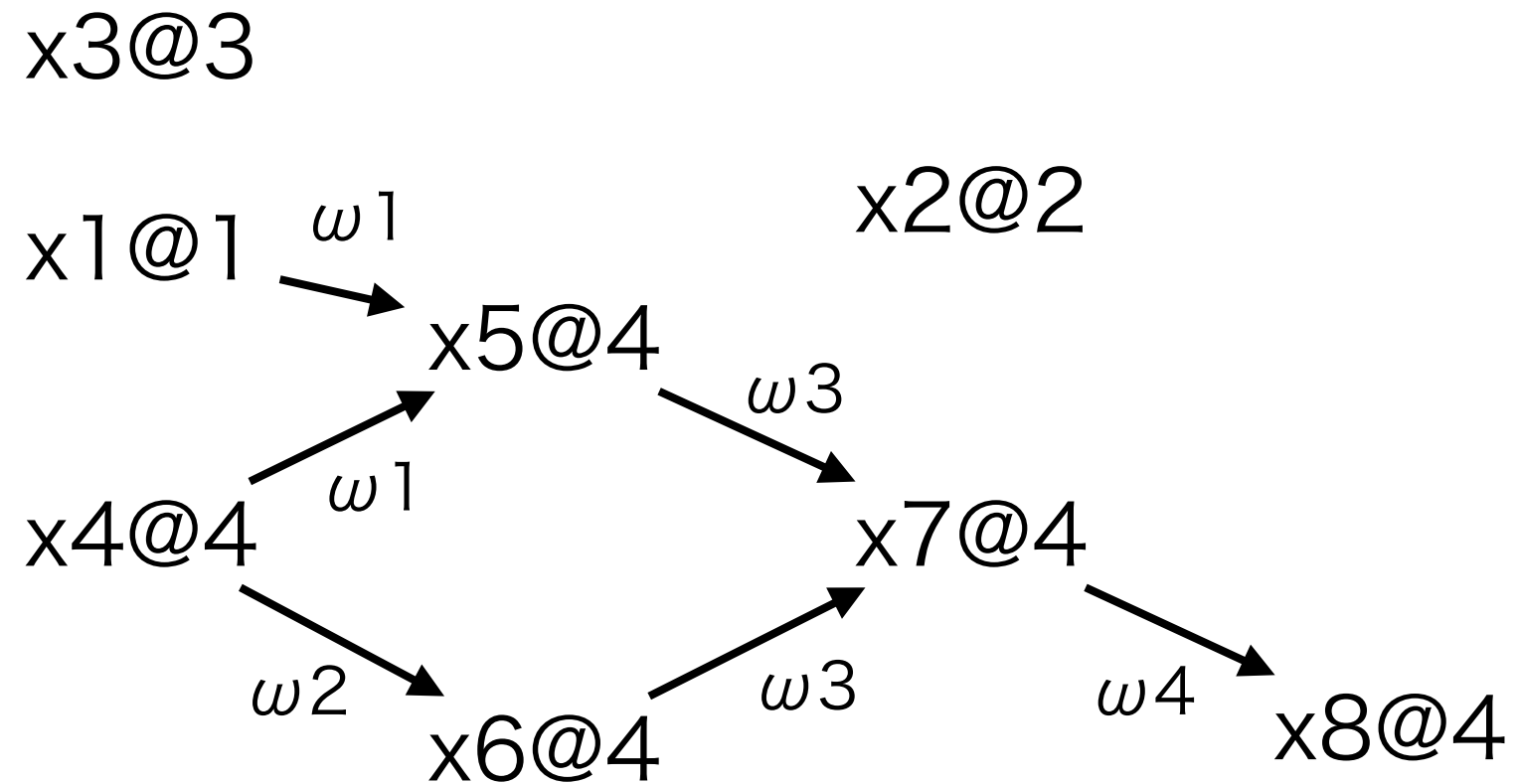
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- $\omega 7: \neg x8 \vee x9$
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Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



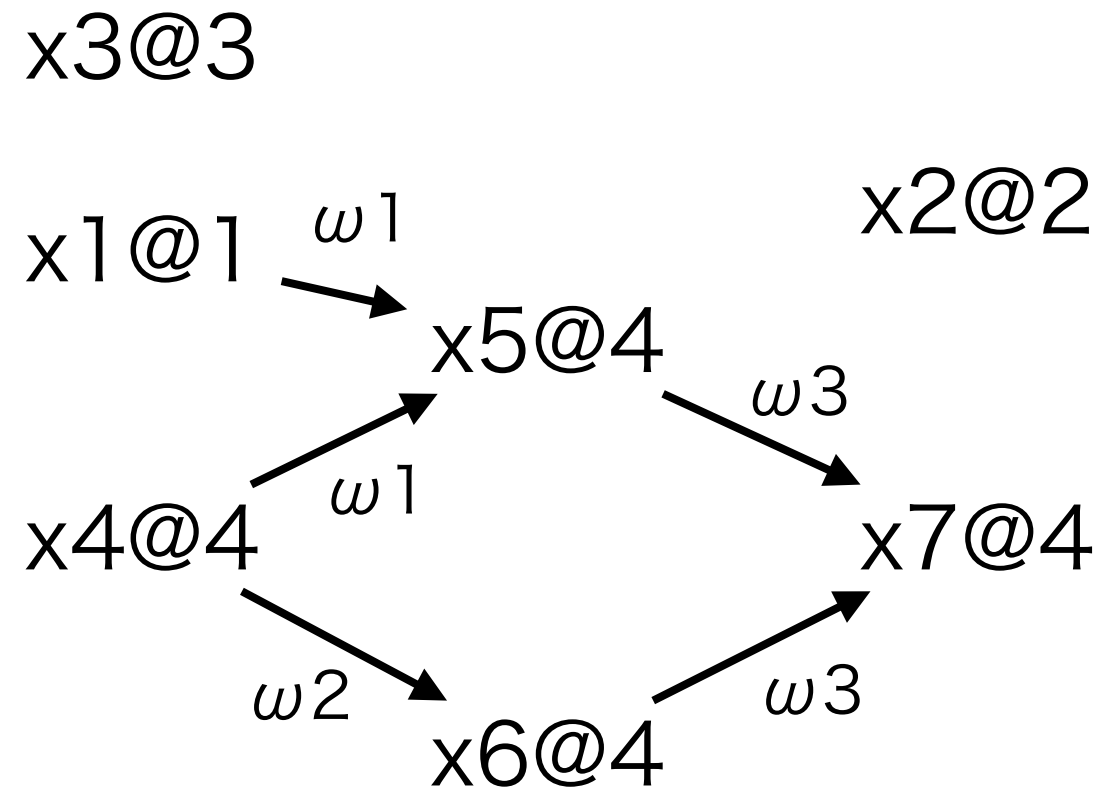
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- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



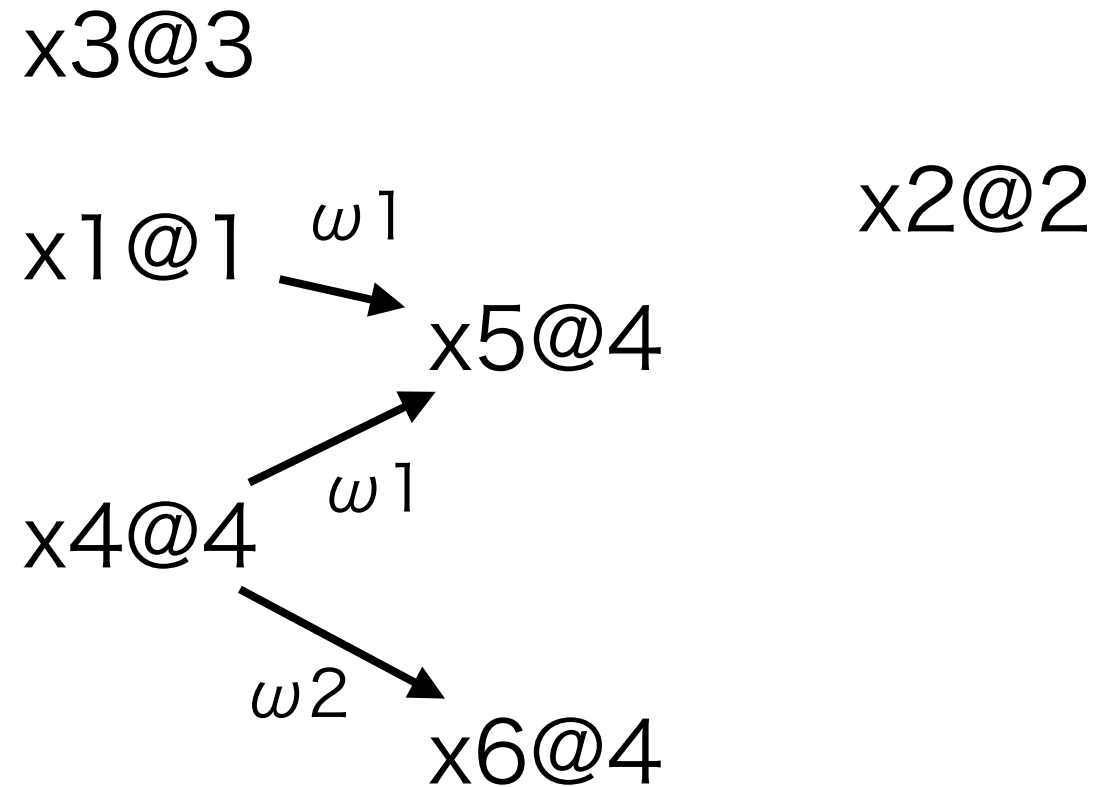
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- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



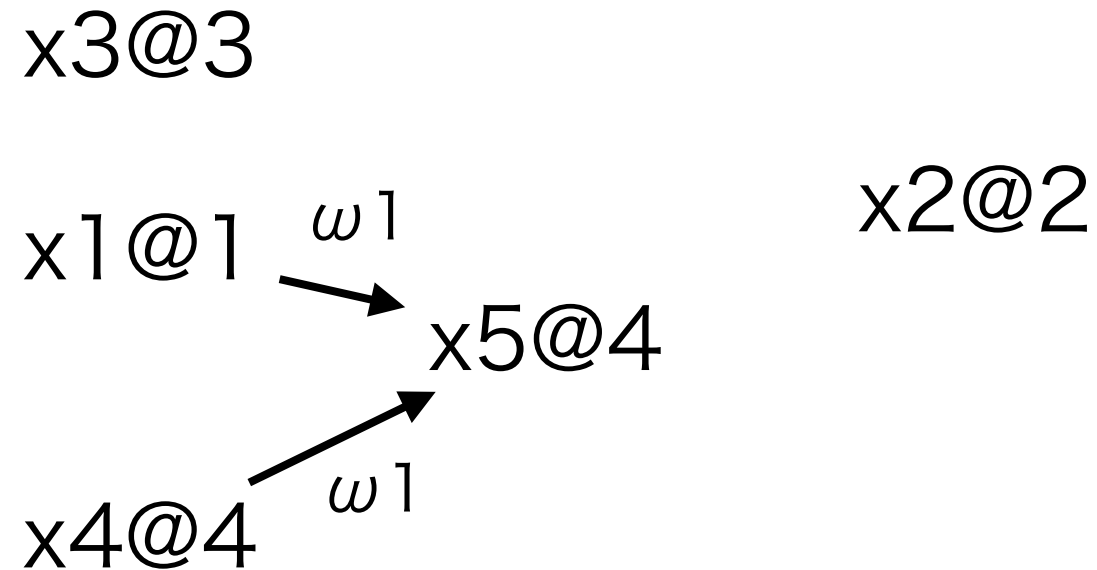
Clause DB

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- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph

x3@3

x1@1

x4@4

x2@2

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
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- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph

x3@3

x1@1

x2@2

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph

x1@1

x2@2

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph

x1@1

x2@2

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph

x1@1

$x2@2 \xrightarrow{\omega 8} \neg x7@2$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Decide

Implication Graph

x1
|
x2
|
x3
|
x4

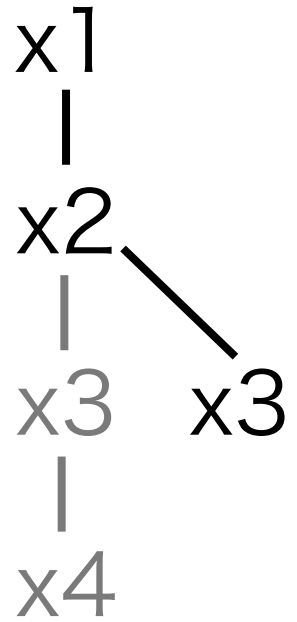
x1@1

$x2@2 \xrightarrow{\omega 8} \neg x7@2$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Decide



Implication Graph

$x3@3$

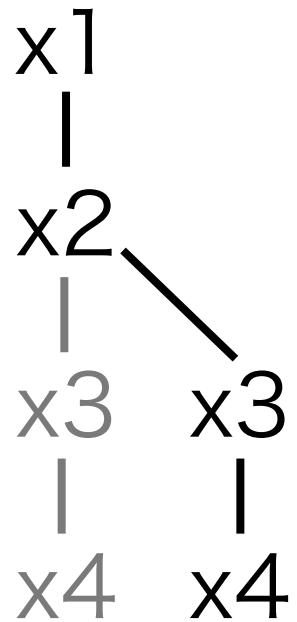
$x1@1$

$x2@2 \xrightarrow{\omega 8} \neg x7@2$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Decide



Implication Graph

$x3@3$

$x1@1$

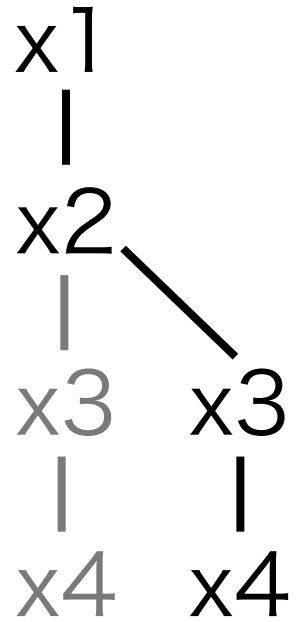
$x4@4$

$x2@2 \xrightarrow{\omega 8} \neg x7@2$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Deduce



Implication Graph

$x3@3$

$x1@1$

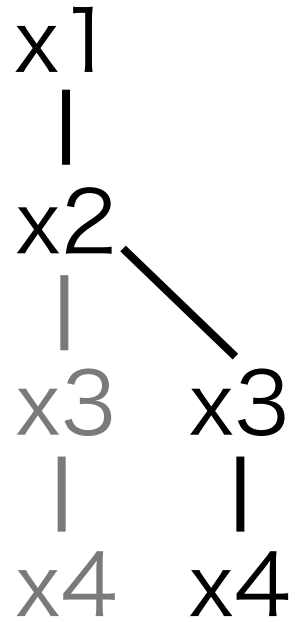
$x4@4$

$x2@2 \xrightarrow{\omega 8} \neg x7@2$

Clause DB

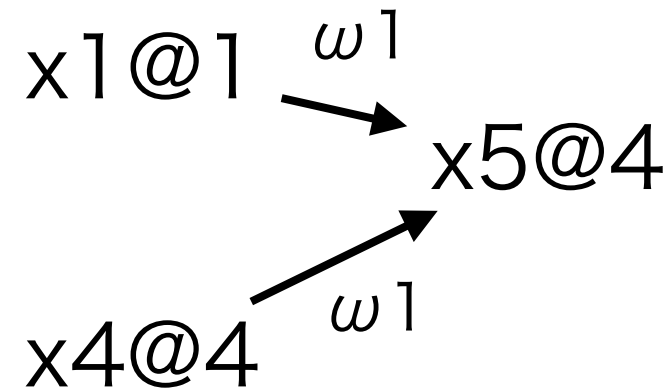
- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Deduce



Implication Graph

$x3@3$

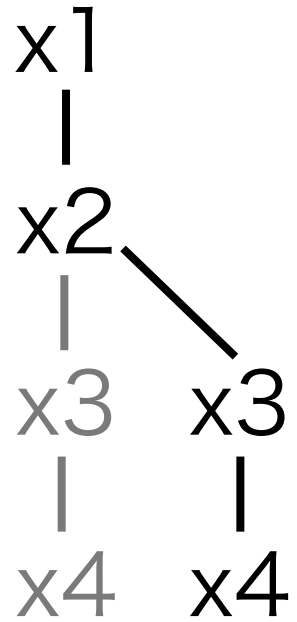


$x2@2 \xrightarrow{\omega 8} \neg x7@2$

Clause DB

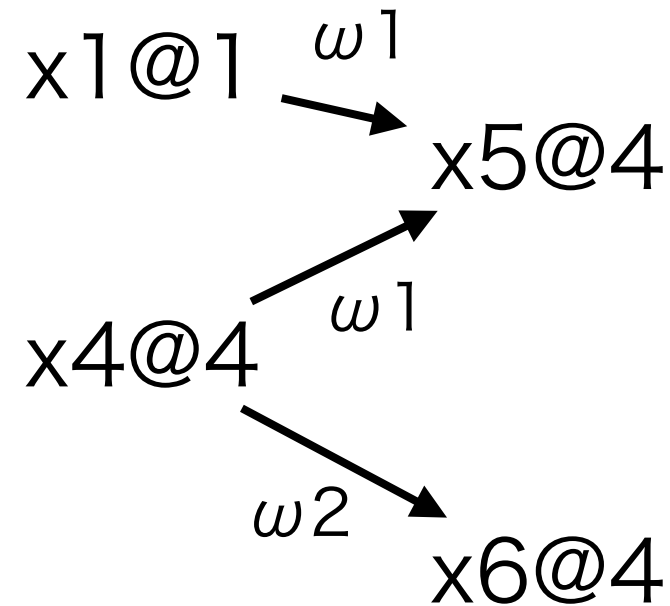
- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Deduce



Implication Graph

$x3@3$

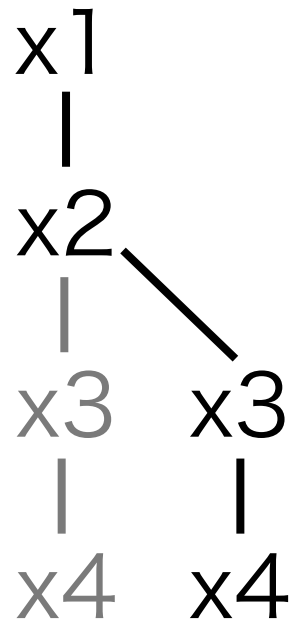


$x2@2 \xrightarrow{\omega 8} \neg x7@2$

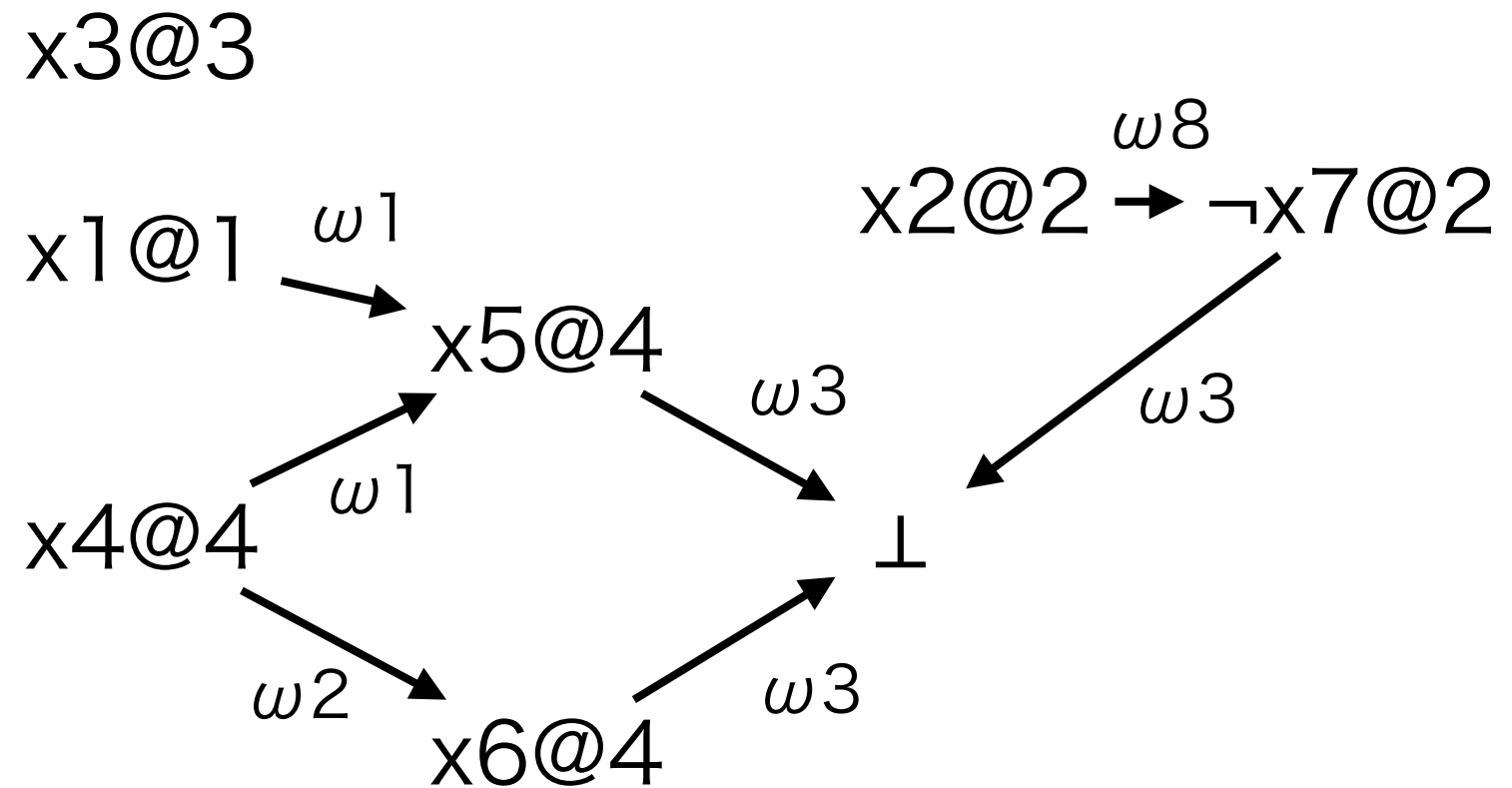
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Deduce



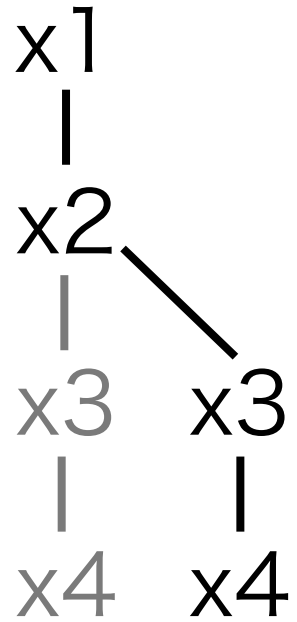
Implication Graph



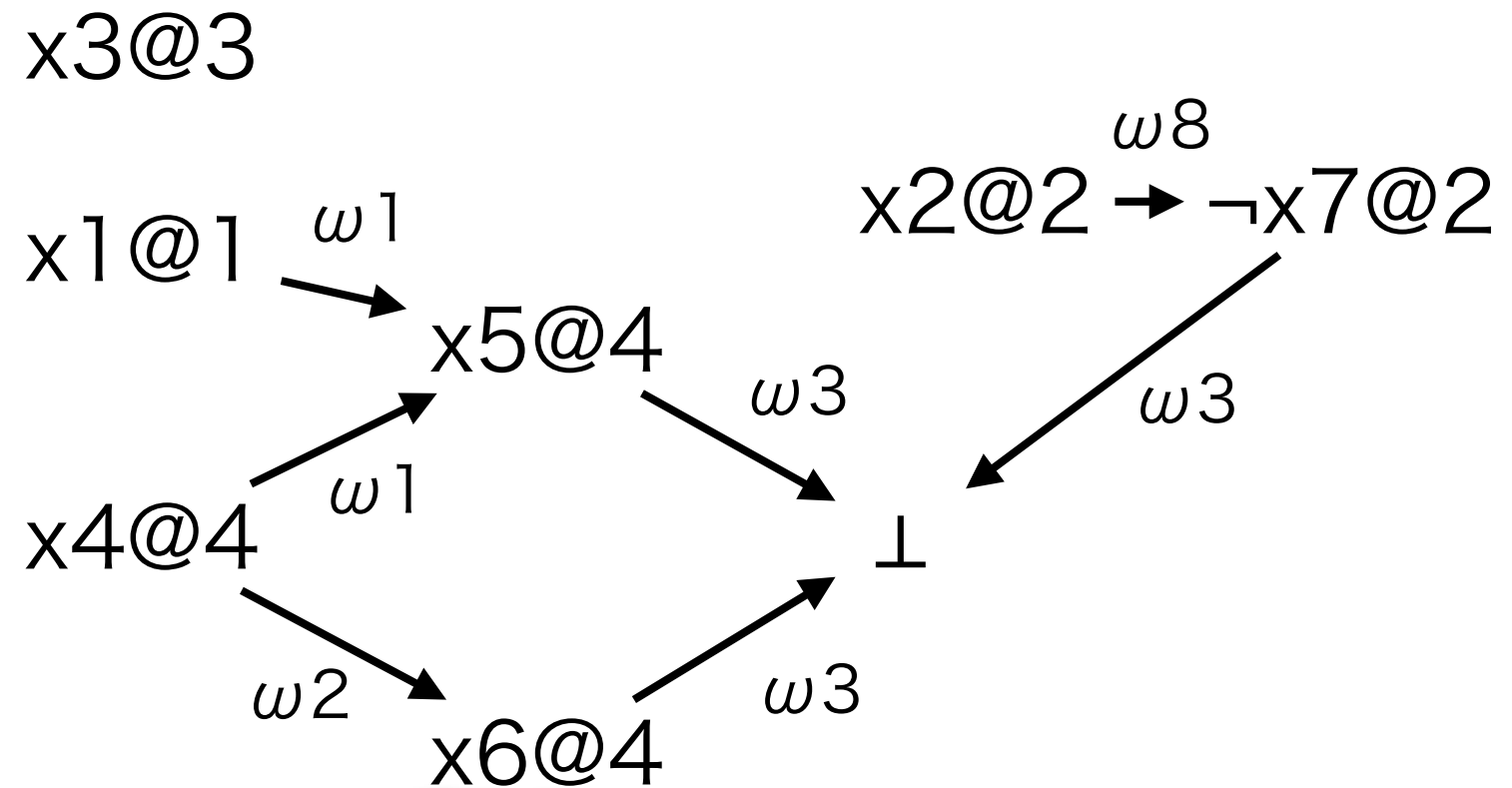
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Deduce



Implication Graph



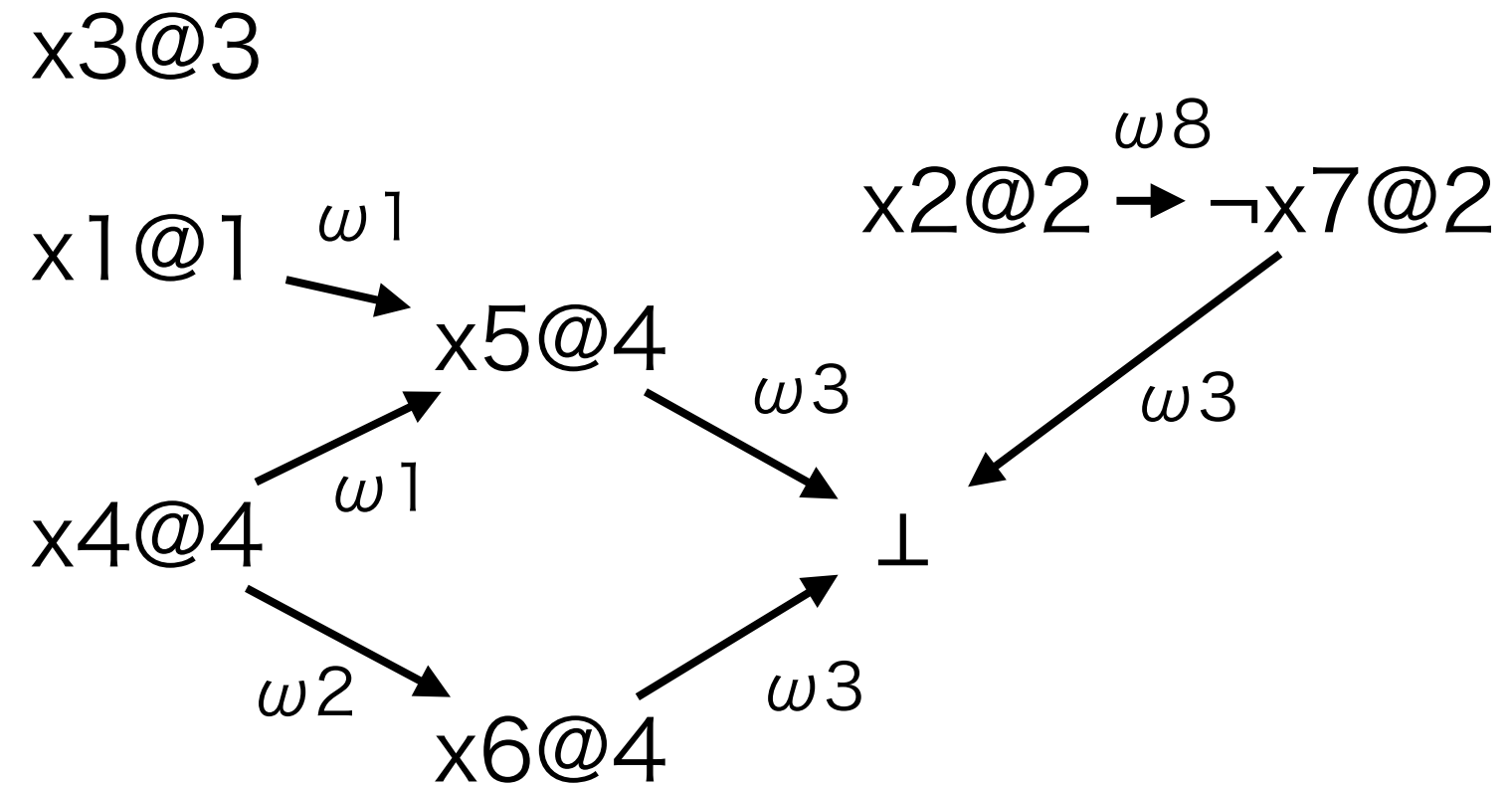
Clause DB

Conflict

- $\omega1: \neg x1 \vee \neg x4 \vee x5$
- $\omega2: \neg x4 \vee x6$
- $\omega3: \neg x5 \vee \neg x6 \vee x7$
- $\omega4: \neg x7 \vee x8$
- $\omega5: \neg x2 \vee \neg x7 \vee x9$
- $\omega6: \neg x8 \vee \neg x9$
- $\omega7: \neg x8 \vee x9$
- $\omega8: \neg x2 \vee \neg x7$

Diagnose

Implication Graph



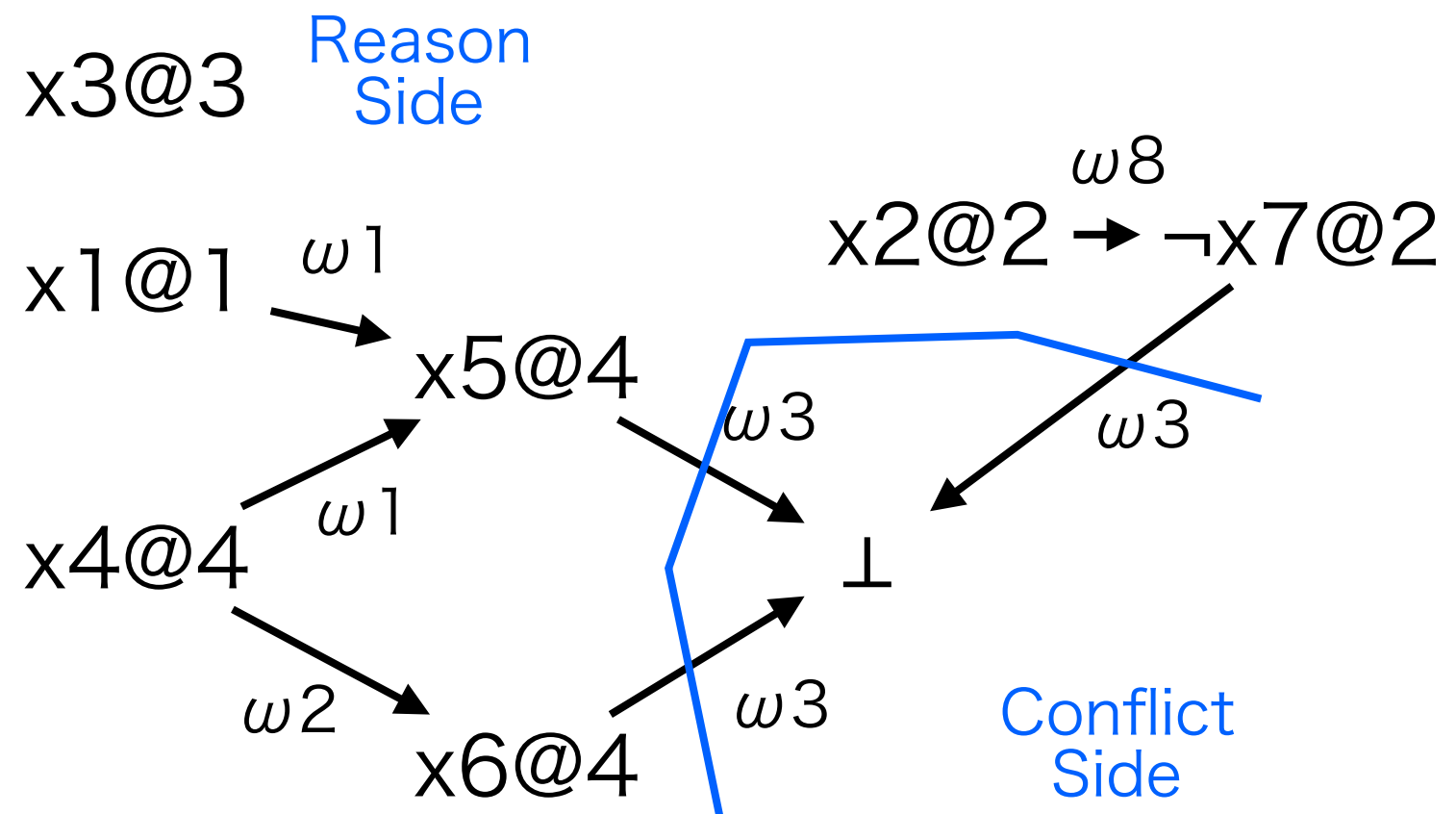
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Diagnose

$\omega 3:$
 $\neg x5 \vee \neg x6 \vee x7$

Implication Graph



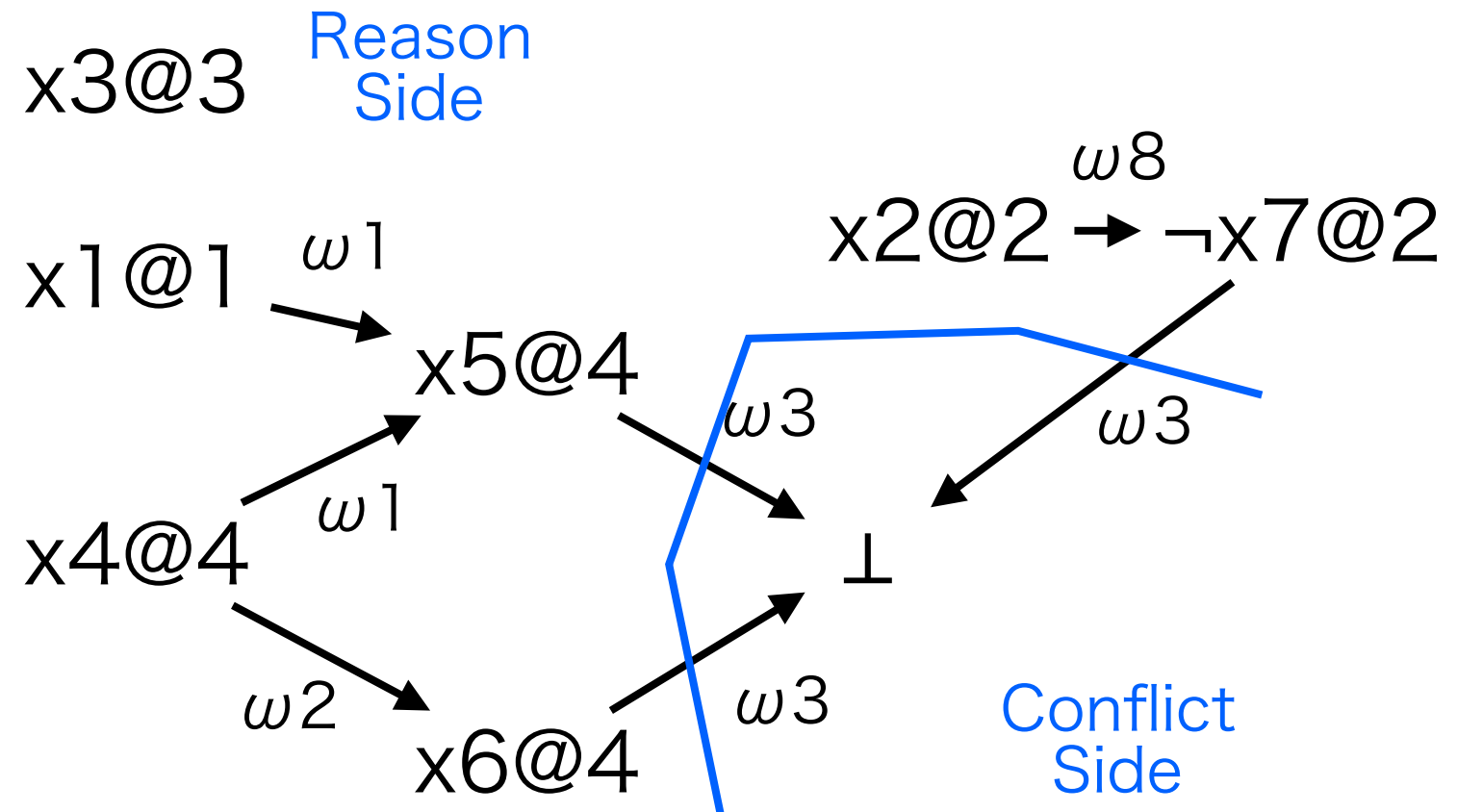
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Diagnose

$\omega 3: \neg x5 \vee \neg x6 \vee x7$ $\omega 2: \neg x4 \vee x6$

Implication Graph



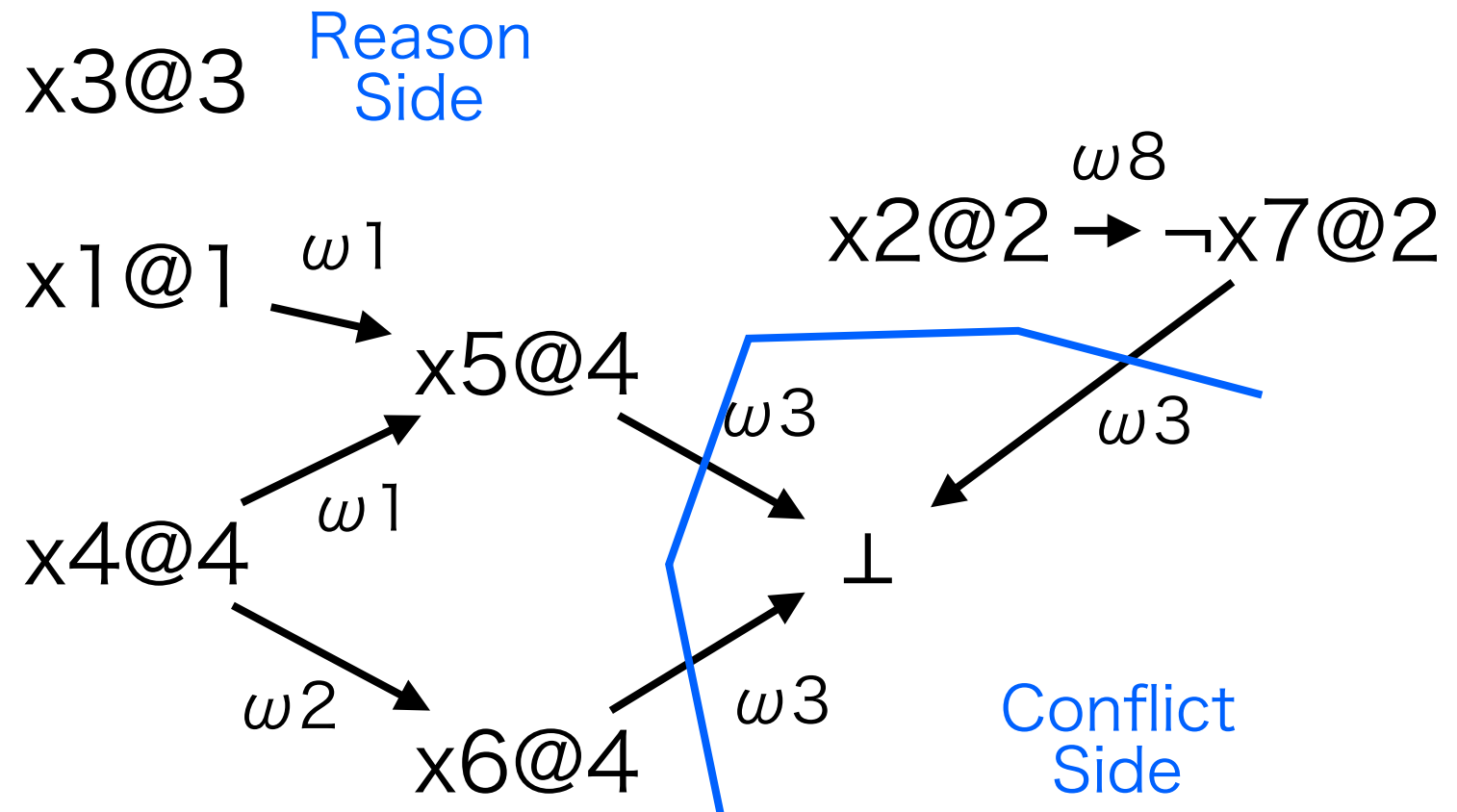
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Diagnose

$\omega 3: \neg x5 \vee \neg x6 \vee x7$ $\omega 2: \neg x4 \vee x6$

Implication Graph



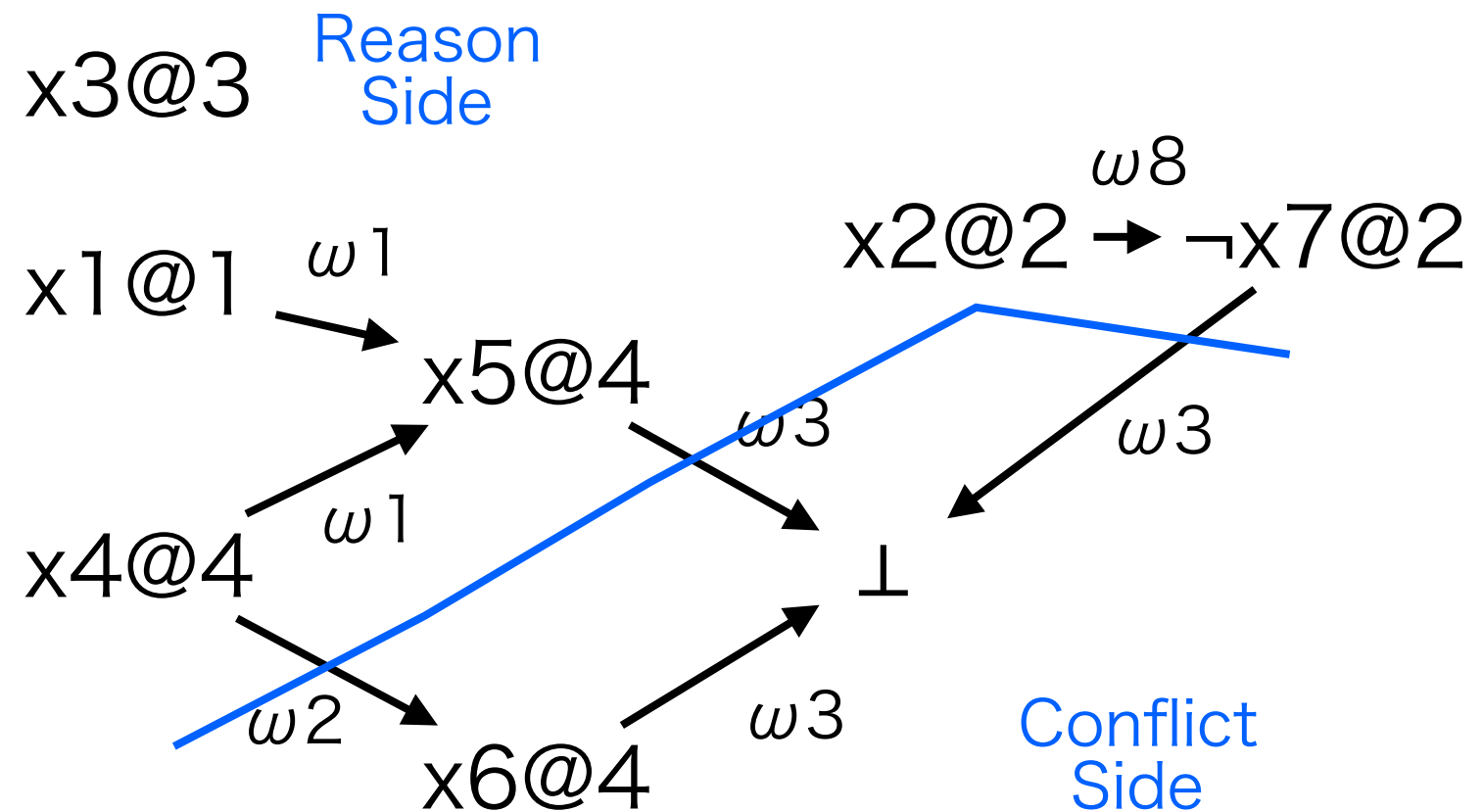
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Diagnose

$$\begin{array}{l} \omega 3: \neg x5 \vee \neg x6 \vee x7 \quad \omega 2: \neg x4 \vee x6 \\ \hline \neg x4 \vee \neg x5 \vee x7 \end{array}$$

Implication Graph



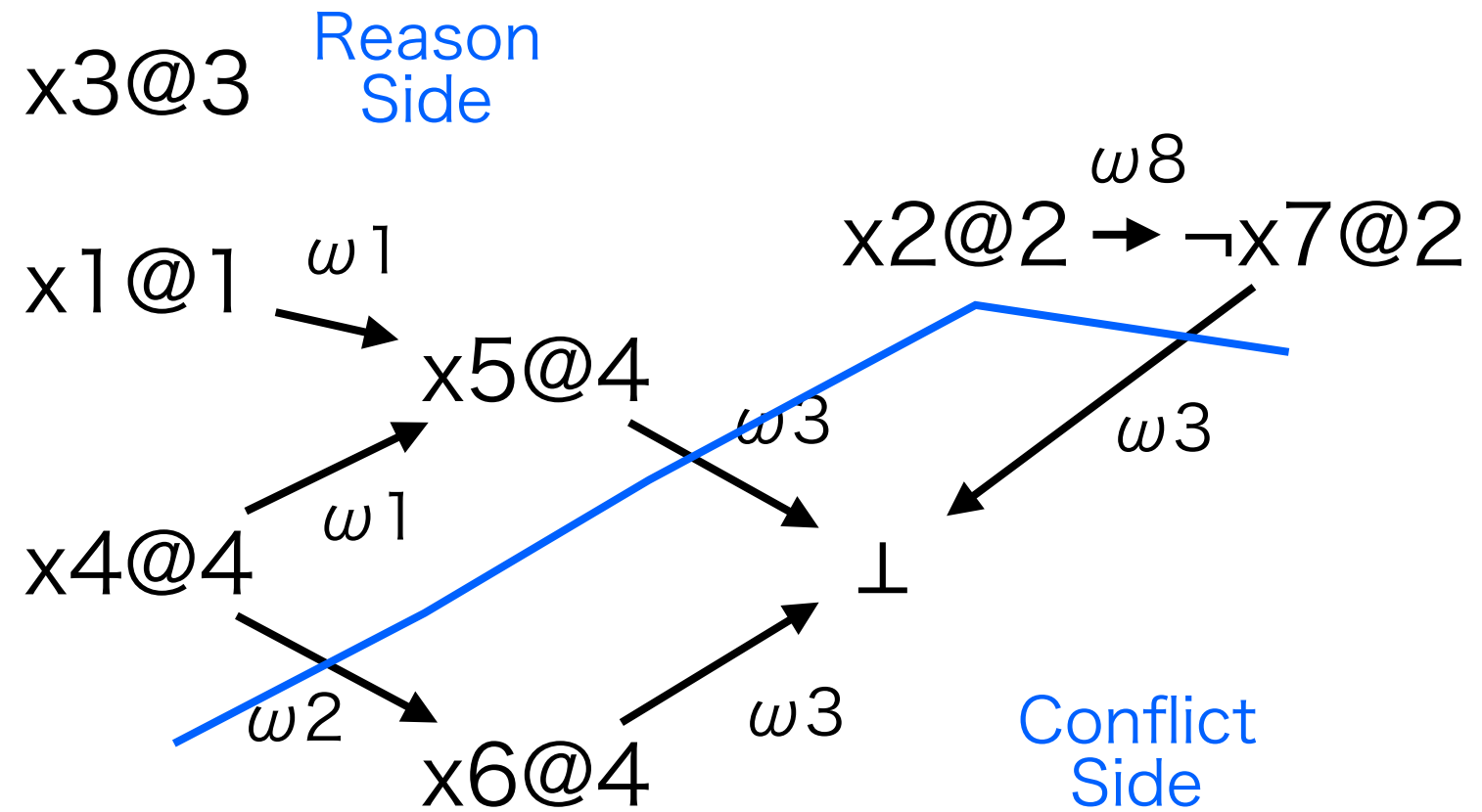
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Diagnose

$$\begin{array}{l} \omega 3: \neg x5 \vee \neg x6 \vee x7 \quad \omega 2: \neg x4 \vee x6 \\ \hline \neg x4 \vee \neg x5 \vee x7 \quad \omega 1: \neg x1 \vee \neg x4 \vee x5 \end{array}$$

Implication Graph



Clause DB

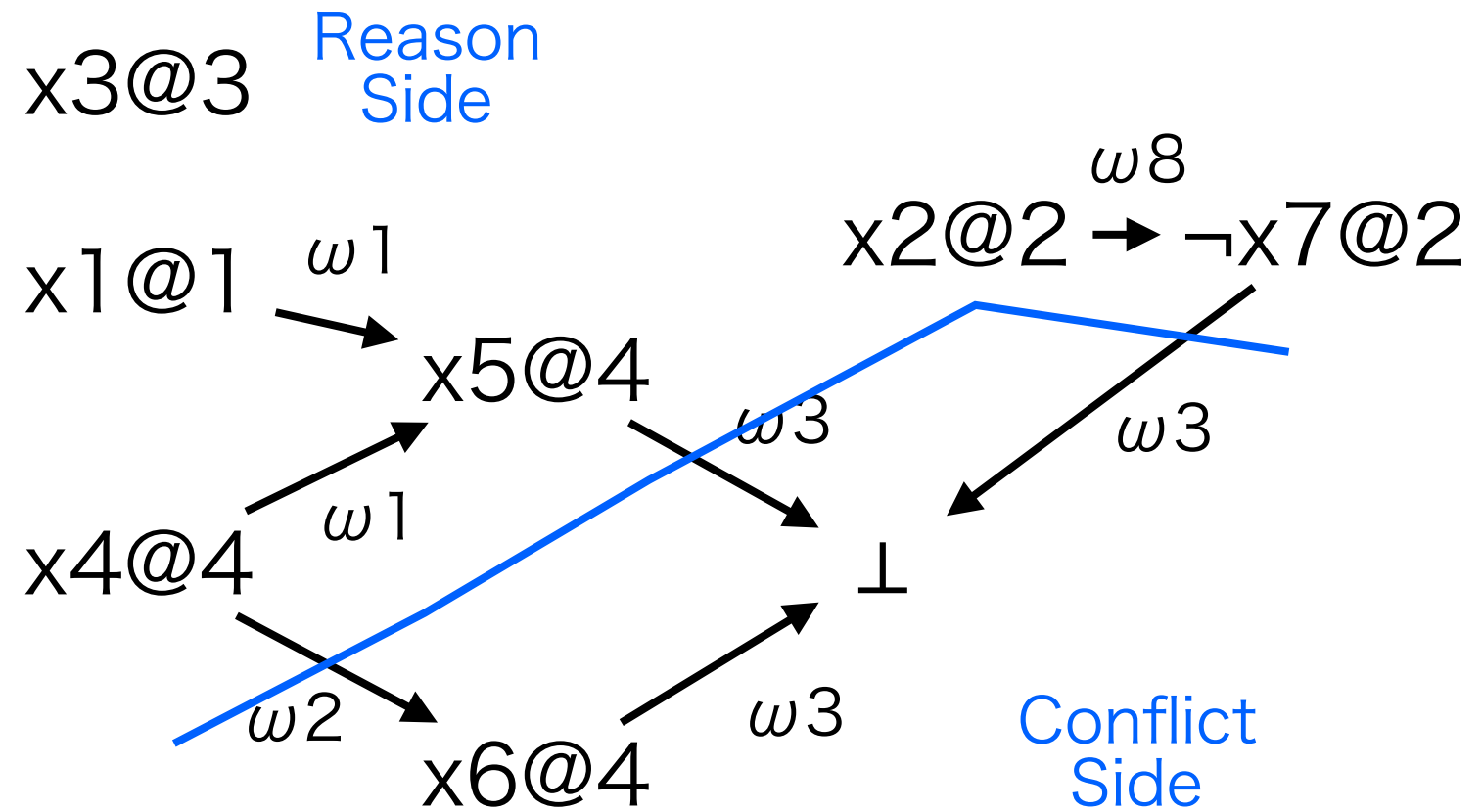
- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Diagnose

$\omega 3: \neg x5 \vee \neg x6 \vee x7$ $\omega 2: \neg x4 \vee x6$

 $\neg x4 \vee \neg x5 \vee x7$ $\omega 1: \neg x1 \vee \neg x4 \vee x5$

Implication Graph



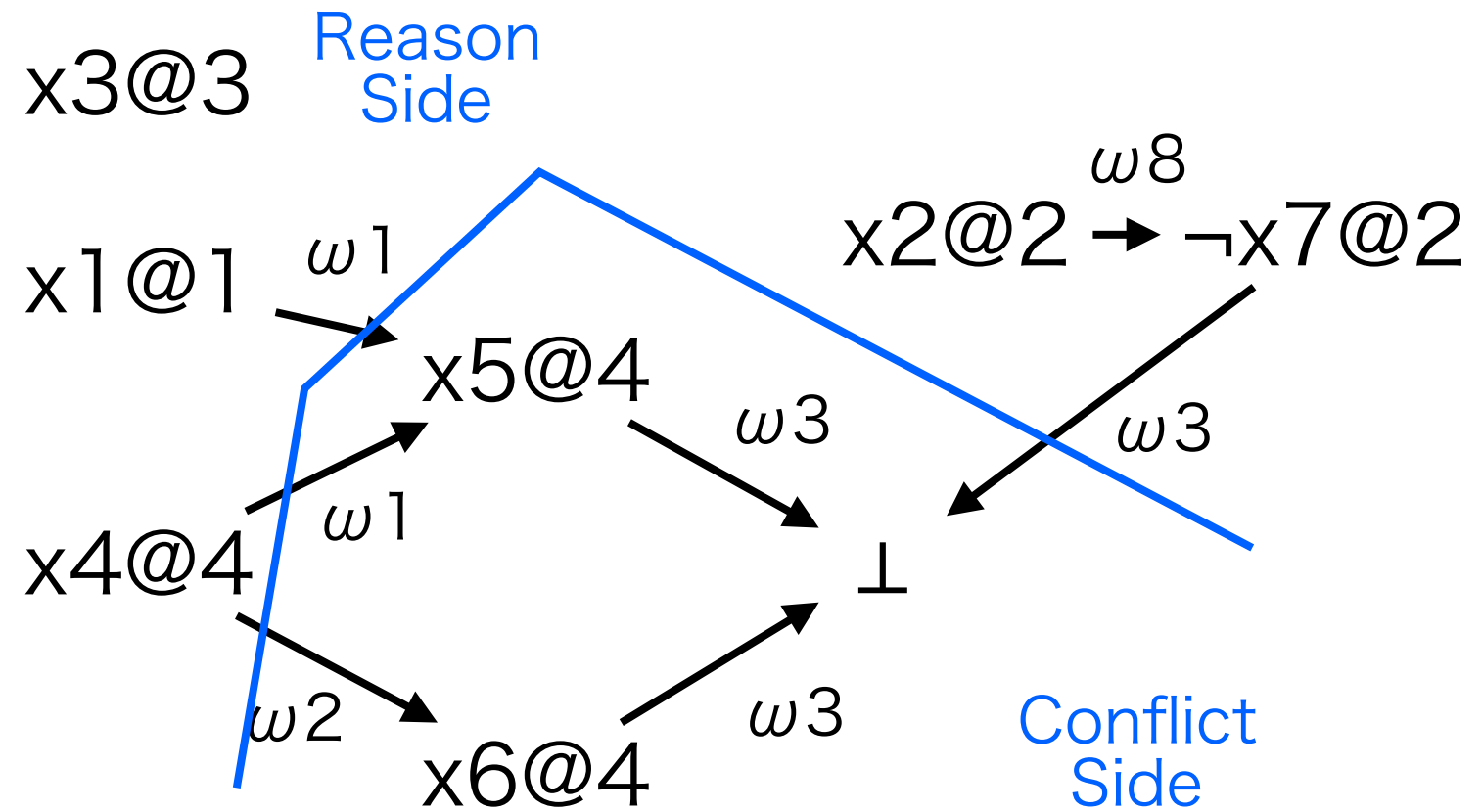
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

Diagnose

$$\begin{array}{c} \omega 3: \quad \omega 2: \\ \neg x 5 \vee \neg x 6 \vee x 7 \quad \neg x 4 \vee x 6 \\ \hline \neg x 4 \vee \neg x 5 \vee x 7 \quad \omega 1: \\ \neg x 1 \vee \neg x 4 \vee x 5 \\ \hline \neg x 1 \vee \neg x 4 \vee x 7 \end{array}$$

Implication Graph



Clause DB

- $\omega 1: \neg x 1 \vee \neg x 4 \vee x 5$
- $\omega 2: \neg x 4 \vee x 6$
- $\omega 3: \neg x 5 \vee \neg x 6 \vee x 7$
- $\omega 4: \neg x 7 \vee x 8$
- $\omega 5: \neg x 2 \vee \neg x 7 \vee x 9$
- $\omega 6: \neg x 8 \vee \neg x 9$
- $\omega 7: \neg x 8 \vee x 9$
- $\omega 8: \neg x 2 \vee \neg x 7$

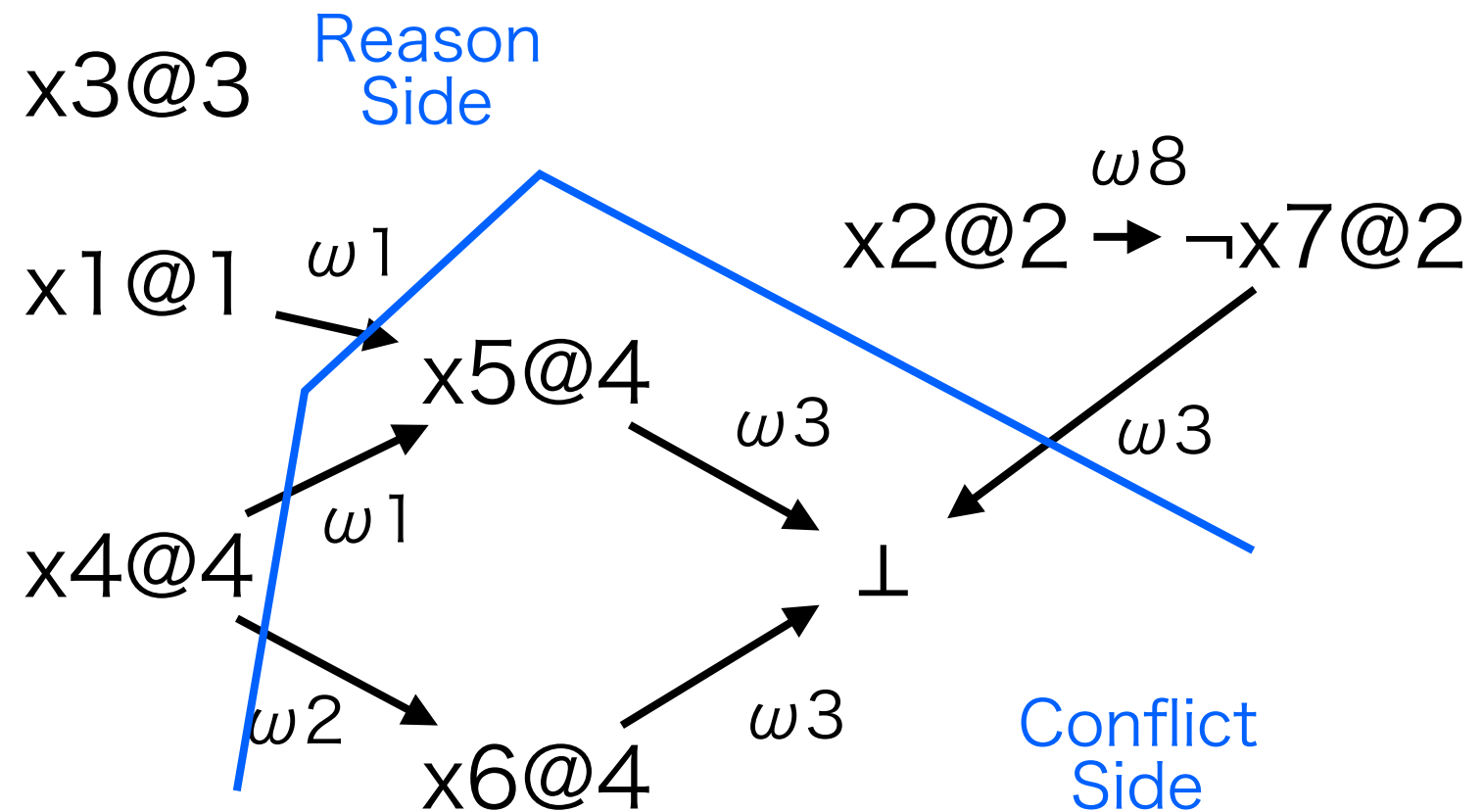
Diagnose

$$\begin{array}{l} \omega 3: \quad \neg x5 \vee \neg x6 \vee x7 \quad \omega 2: \quad \neg x4 \vee x6 \\ \hline \neg x4 \vee \neg x5 \vee x7 \quad \omega 1: \quad \neg x1 \vee \neg x4 \vee x5 \end{array}$$

$$\neg x1 \vee \neg x4 \vee x7$$

It contains one literal
in the current
decision level (i.e. 4).

Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$

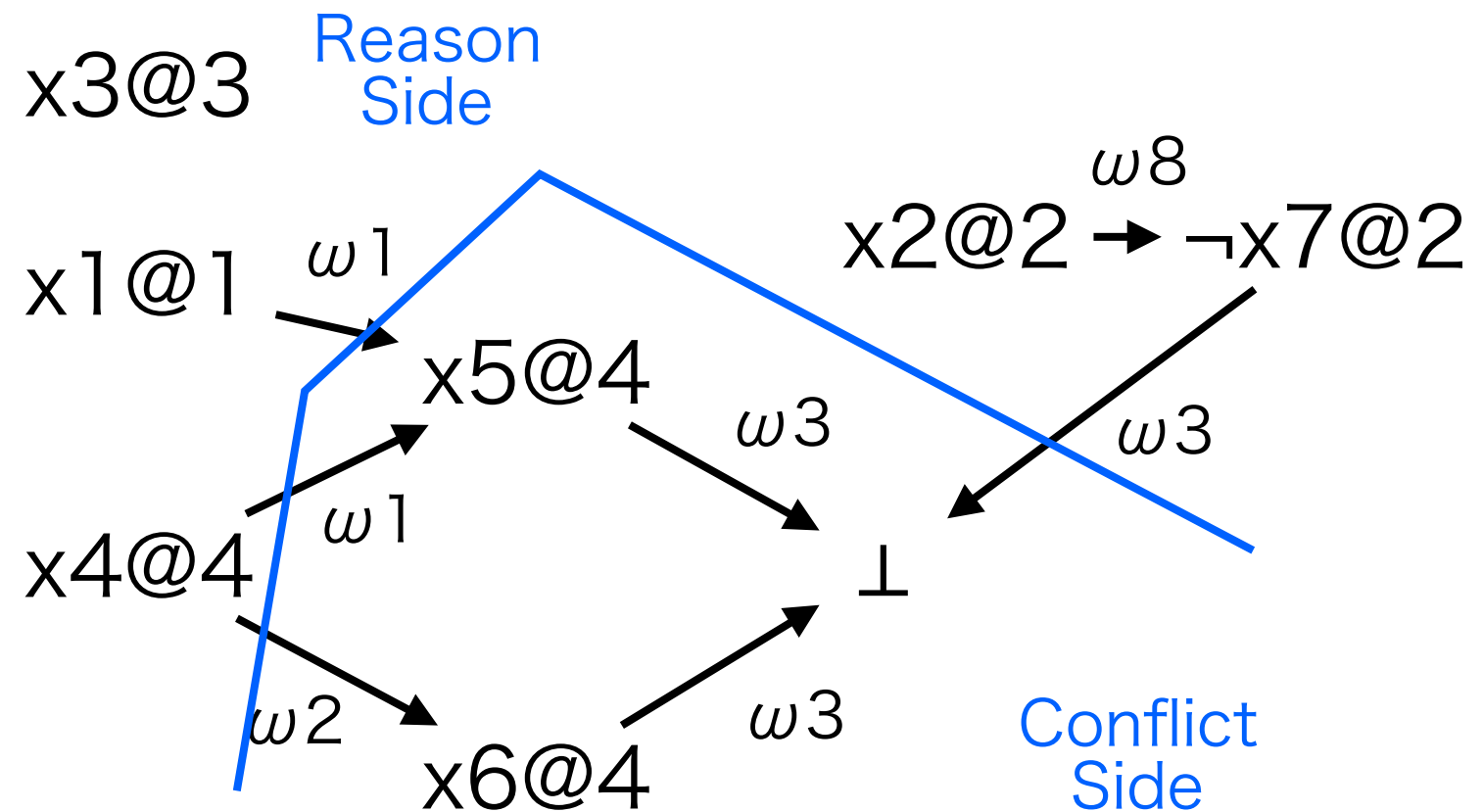
Diagnose

$$\begin{array}{l} \omega 3: \neg x5 \vee \neg x6 \vee x7 \quad \omega 2: \neg x4 \vee x6 \\ \hline \neg x4 \vee \neg x5 \vee x7 \quad \omega 1: \neg x1 \vee \neg x4 \vee x5 \\ \hline \end{array}$$

$$\neg x1 \vee \neg x4 \vee x7$$

It contains one literal
in the current
decision level (i.e. 4).

Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

New clause is learnt !

Backtrack

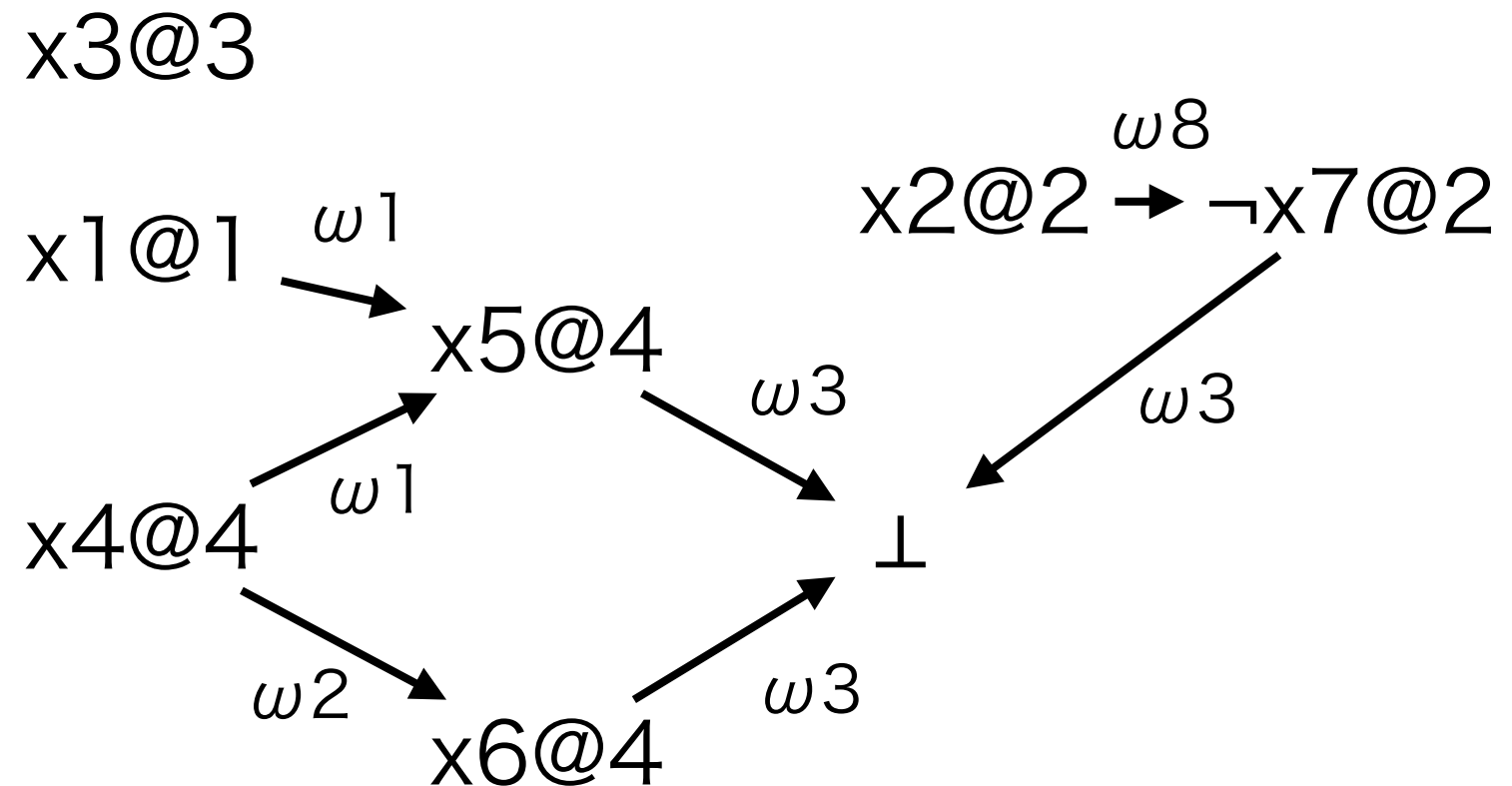
Learnt clause:
 $\neg x1 \vee \neg x4 \vee x7$

x1 was set at level 1
x4 was set at level 4
x7 was set at level 2

Assertion level of this
clause is 2.

$\Rightarrow$ Backtrack to level 2!

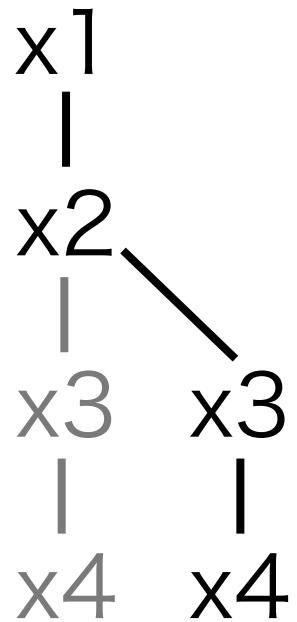
Implication Graph



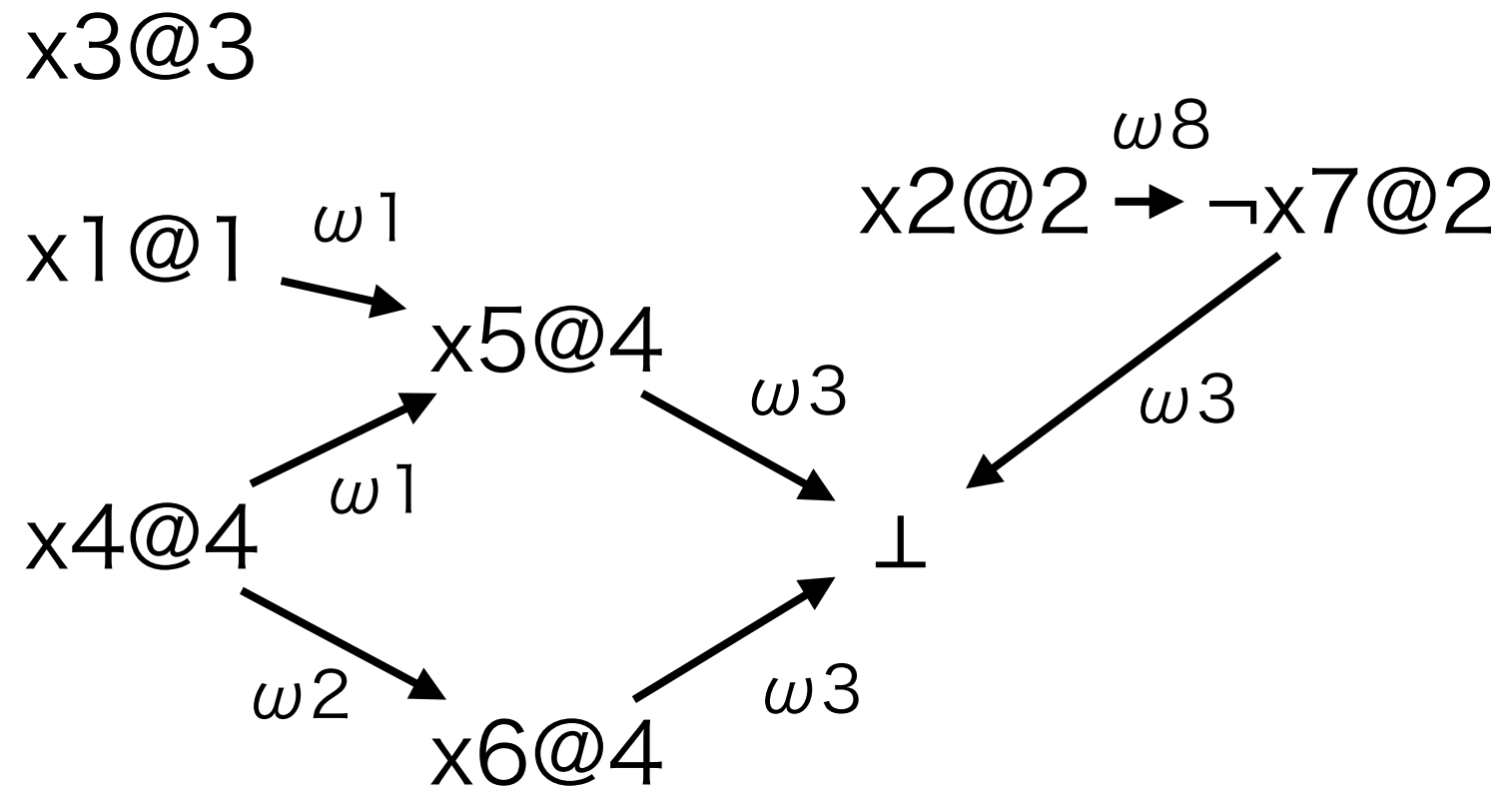
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Backtrack



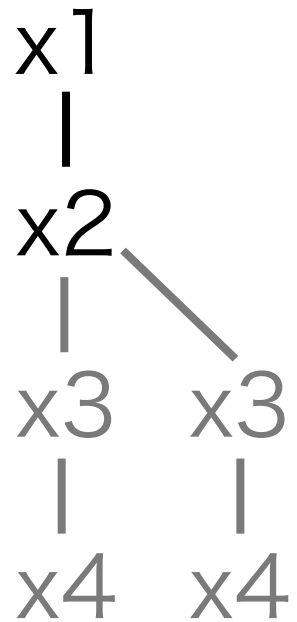
Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Backtrack



Implication Graph

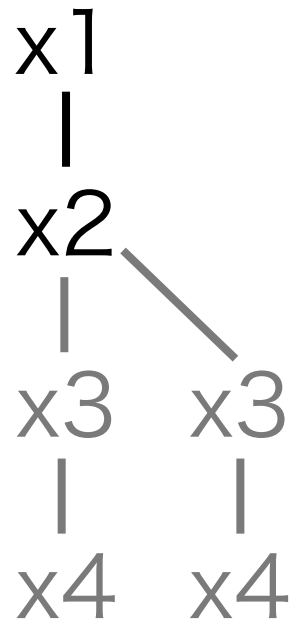
$x1@1$

$x2@2 \xrightarrow{\omega 8} \neg x7@2$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Deduce



Implication Graph

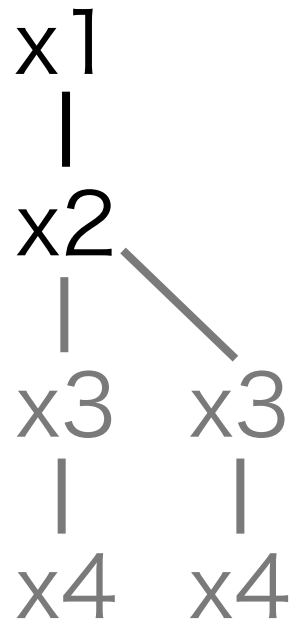
$x1@1$

$x2@2 \xrightarrow{\omega 8} \neg x7@2$

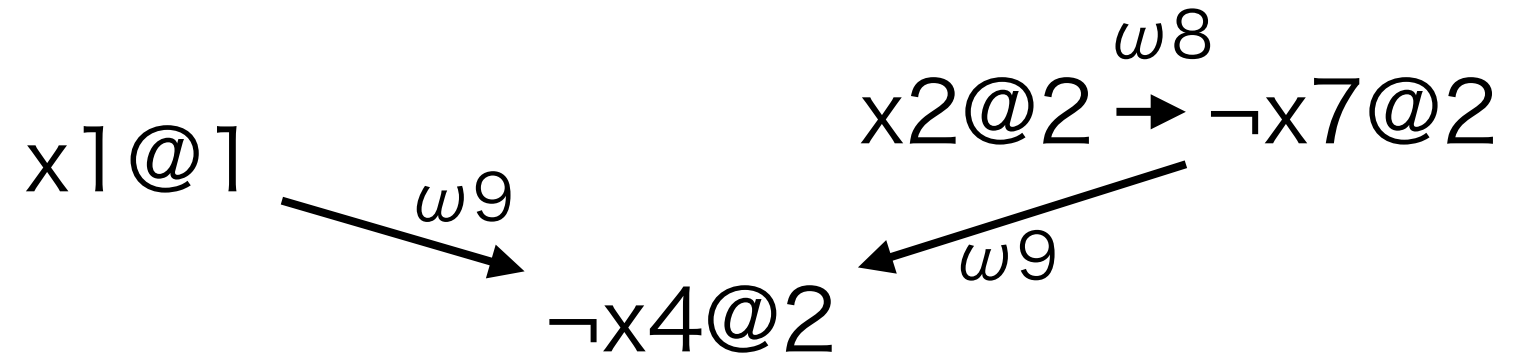
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Deduce



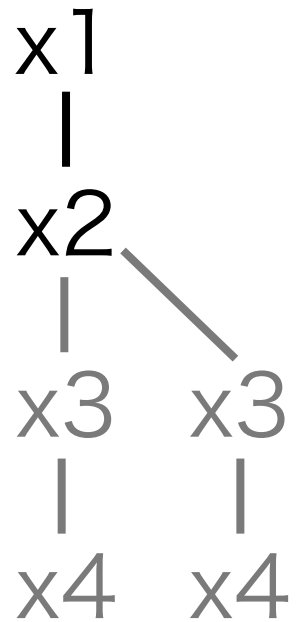
Implication Graph



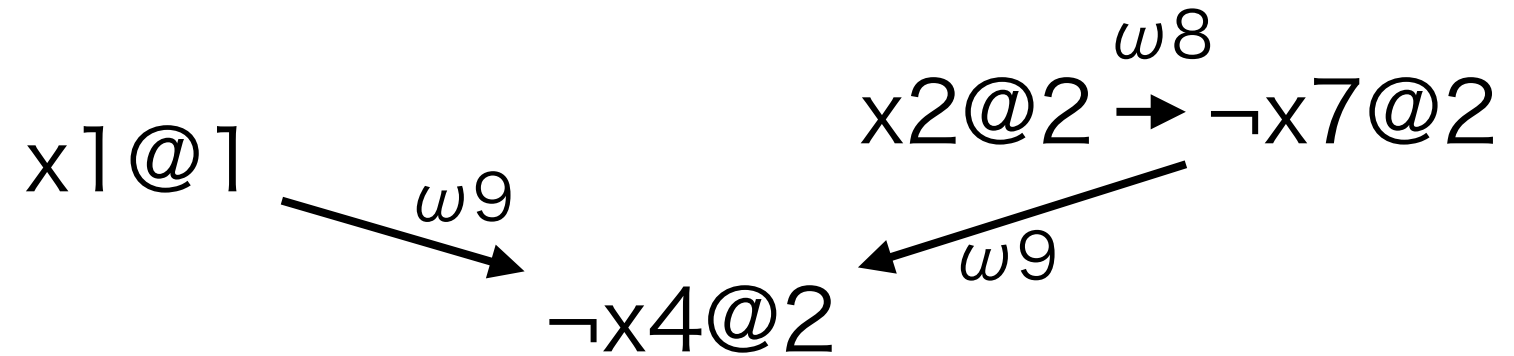
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Decide



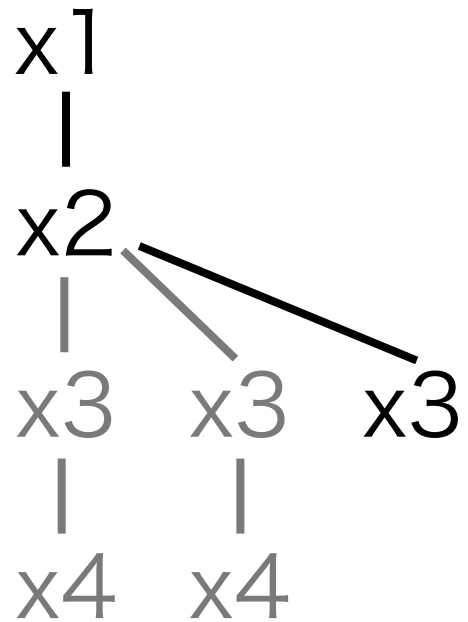
Implication Graph



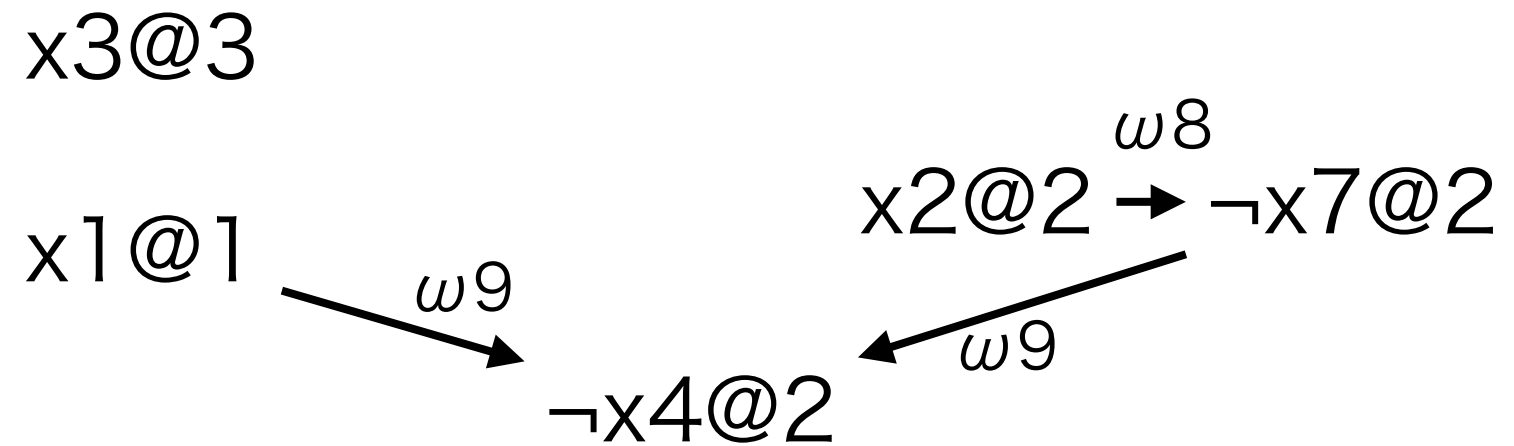
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Decide



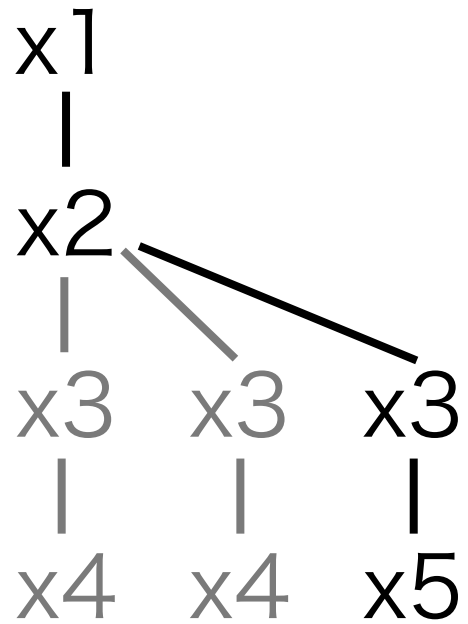
Implication Graph



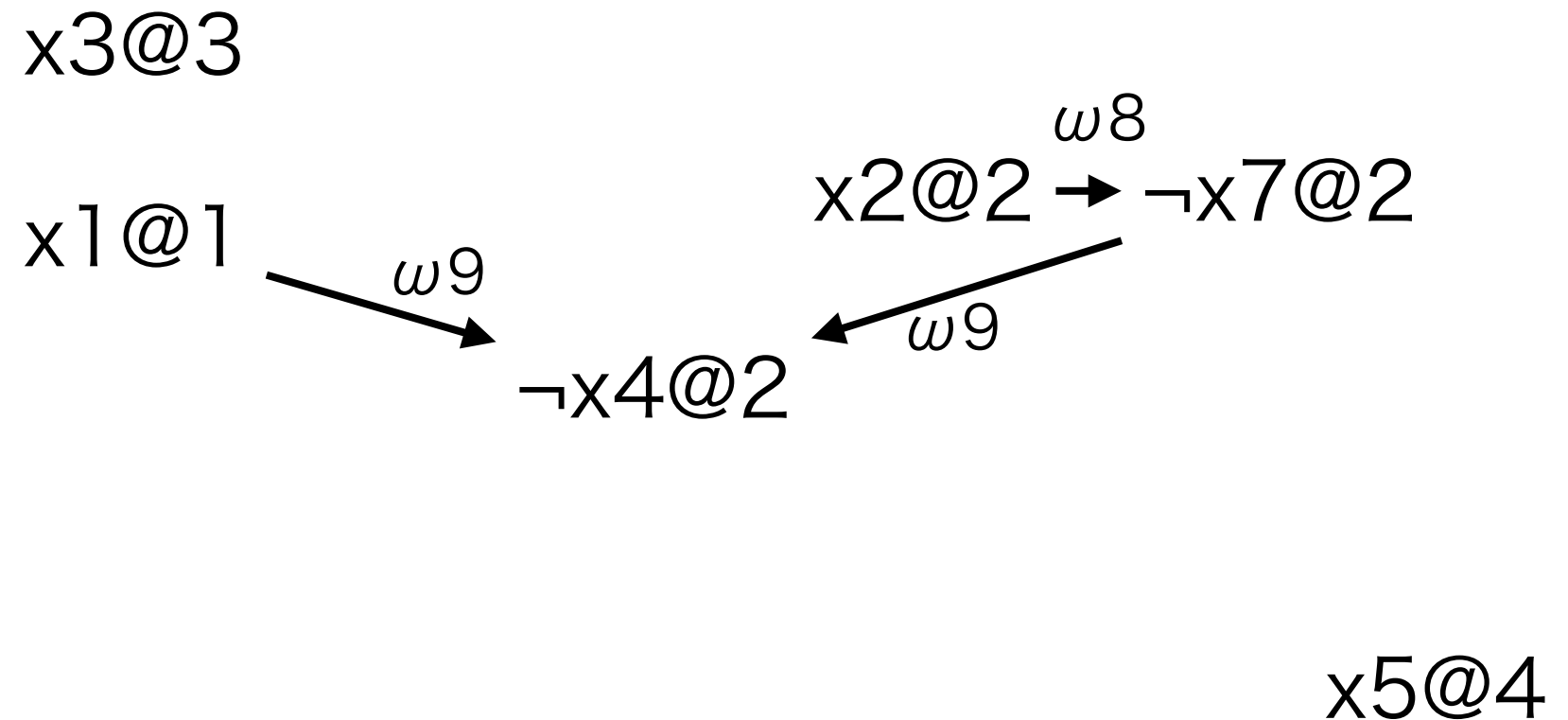
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Decide



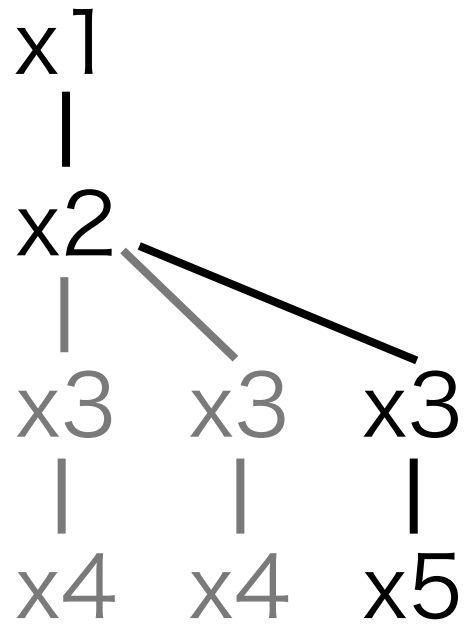
Implication Graph



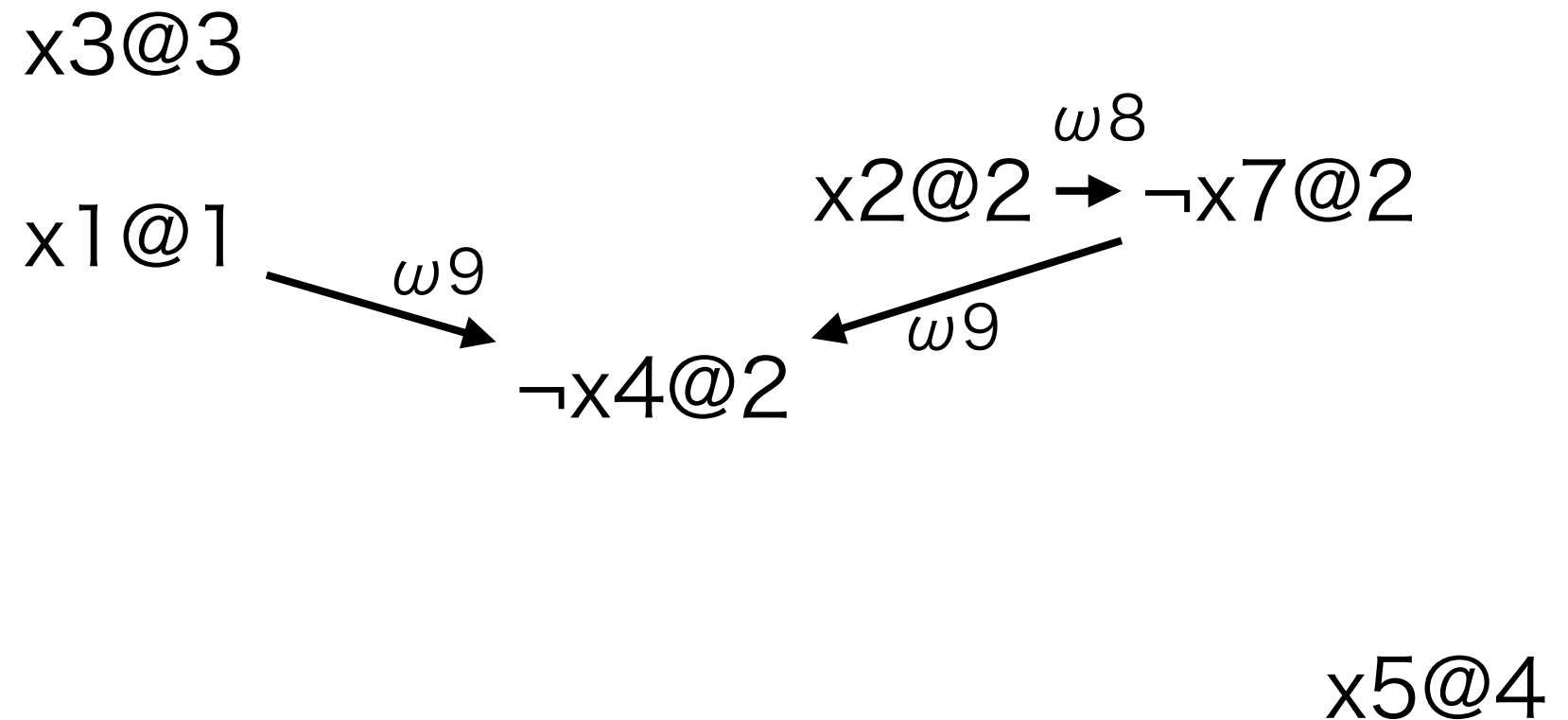
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
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- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Deduce



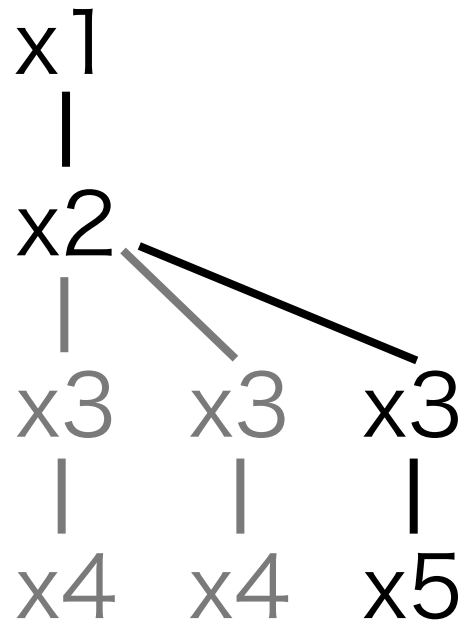
Implication Graph



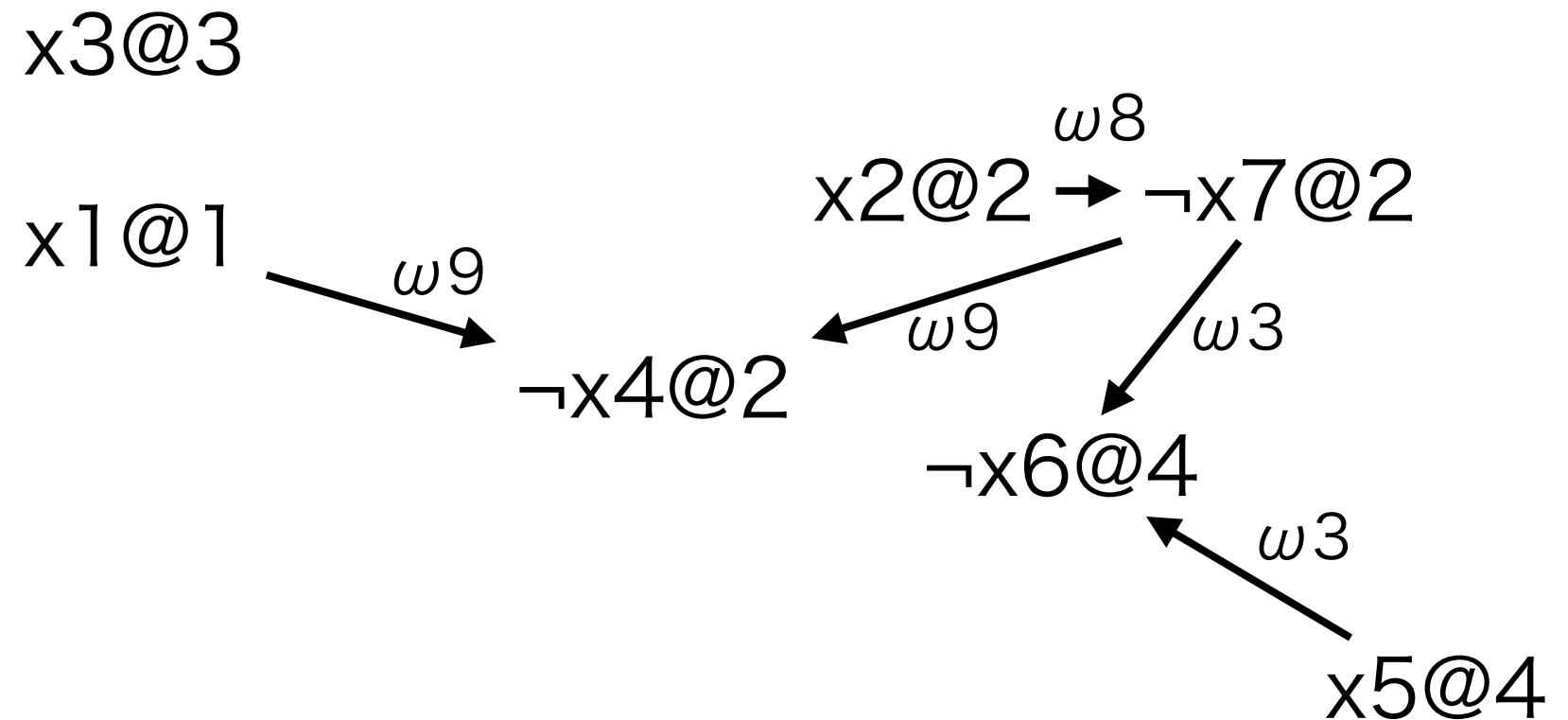
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Deduce



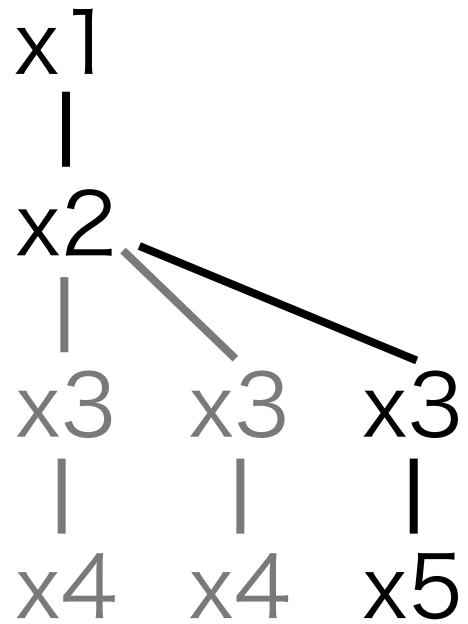
Implication Graph



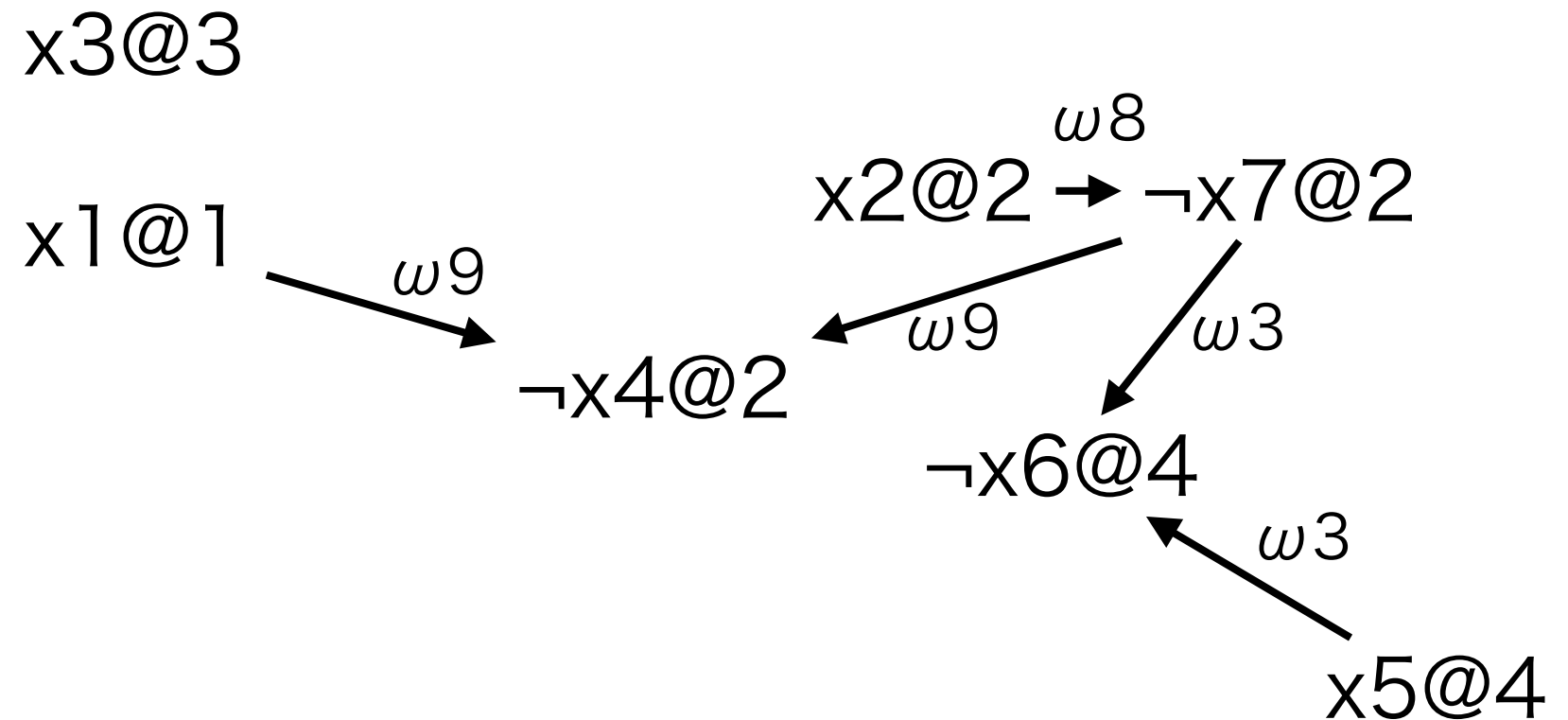
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
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- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Decide



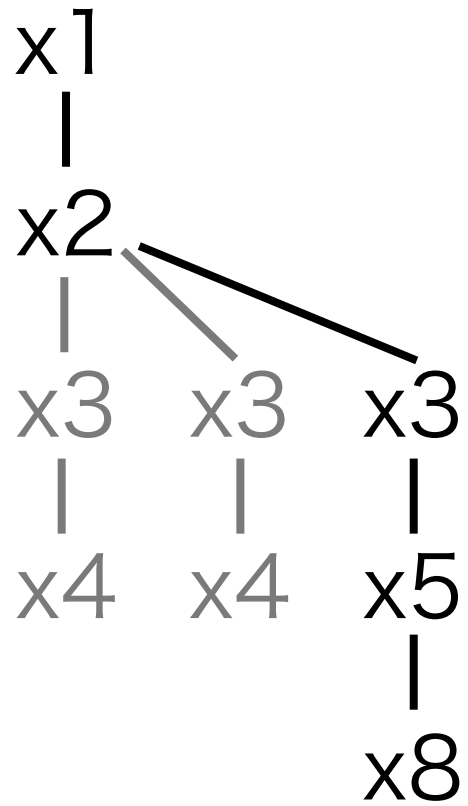
Implication Graph



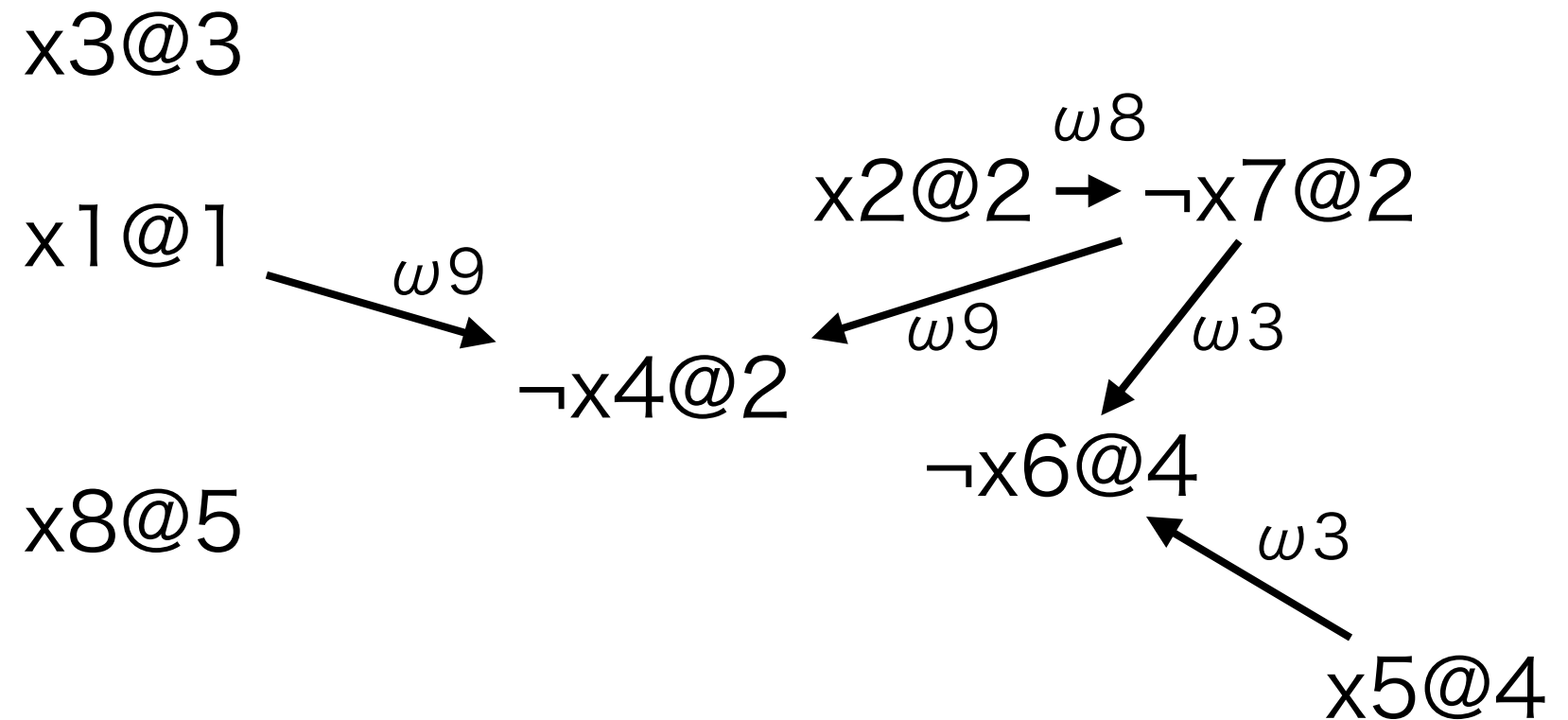
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
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- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Decide



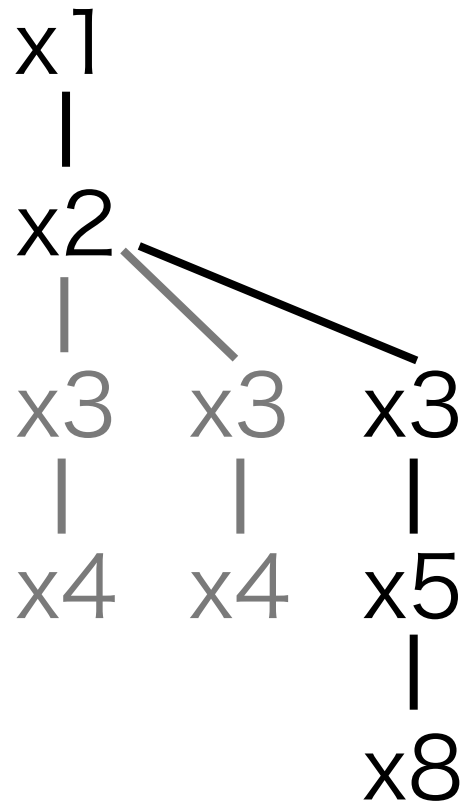
Implication Graph



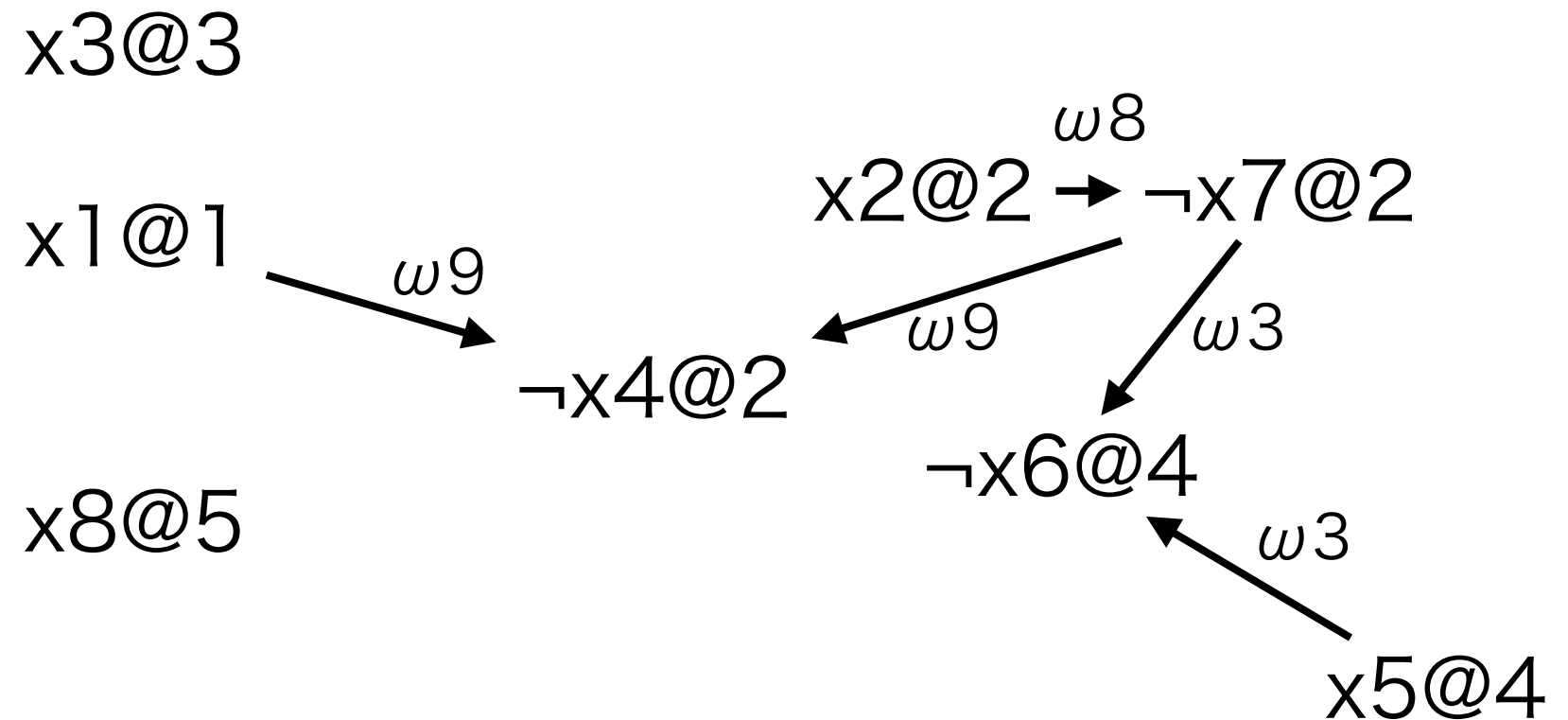
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- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
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- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
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- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Deduce



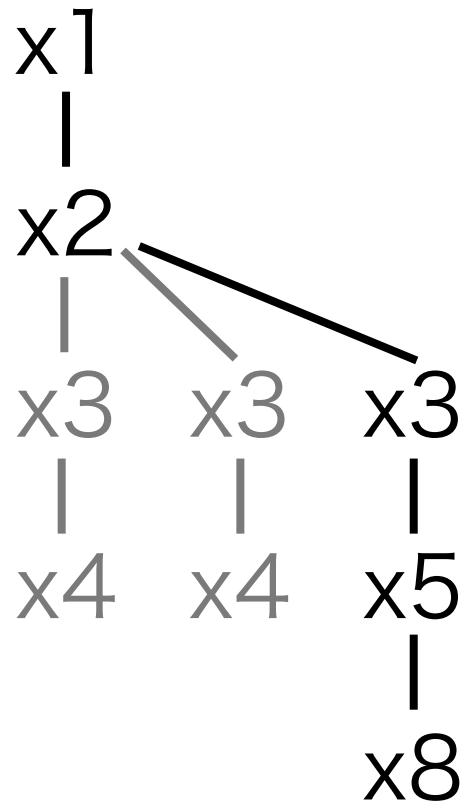
Implication Graph



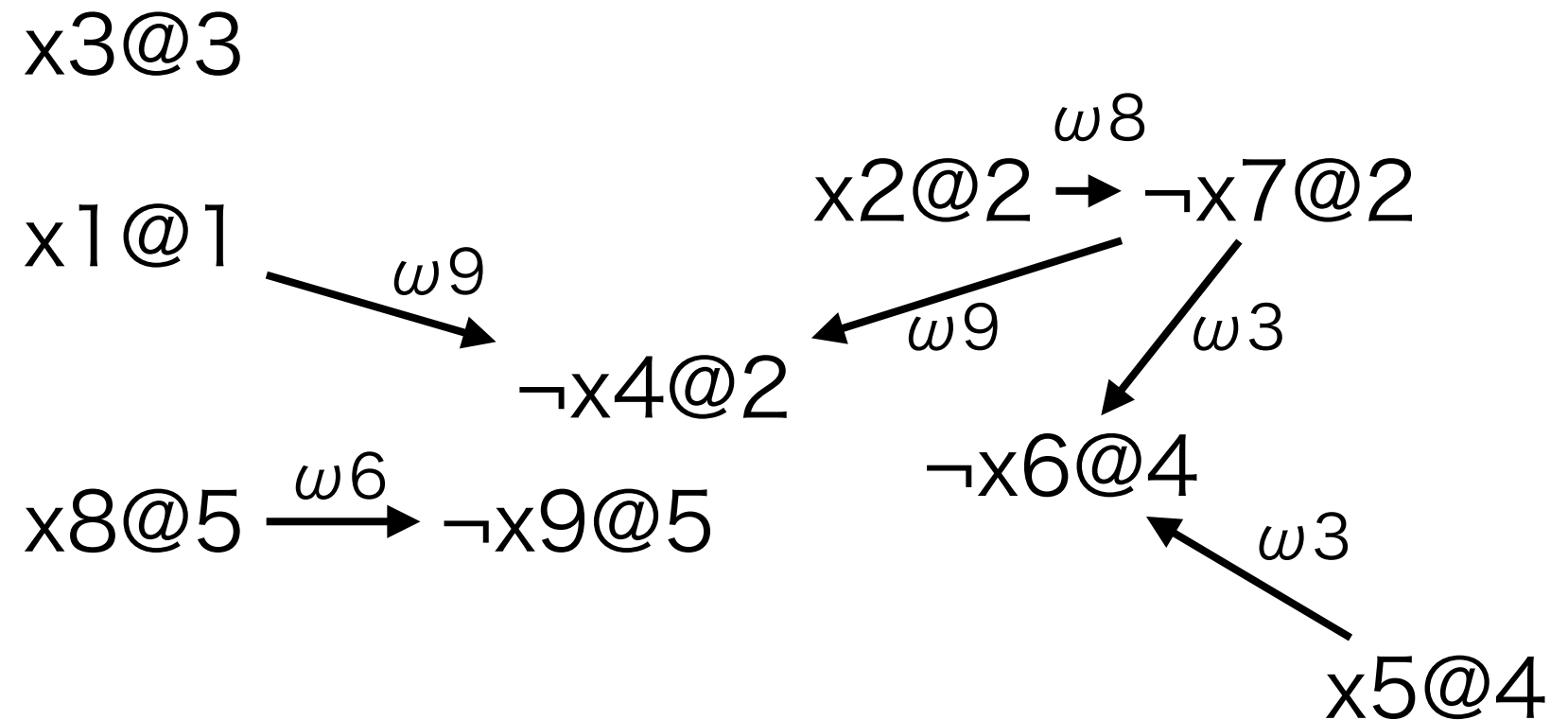
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Deduce



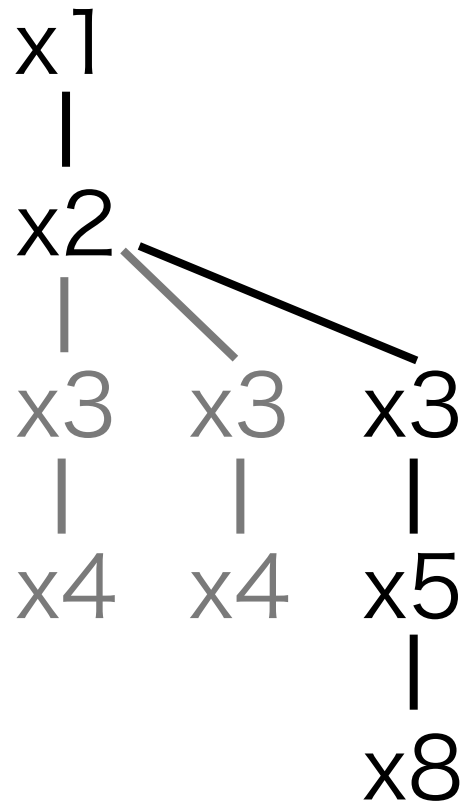
Implication Graph



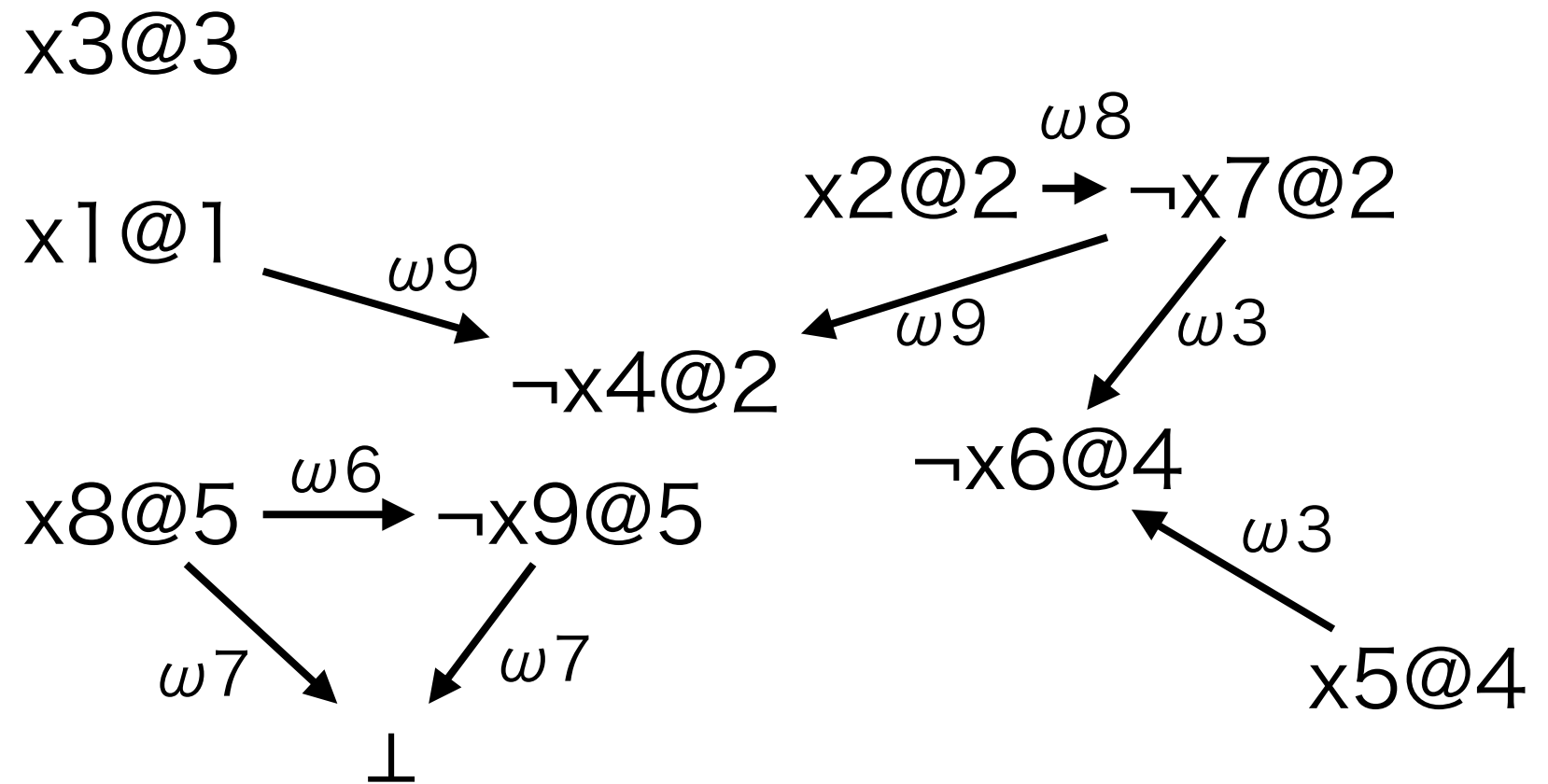
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Deduce



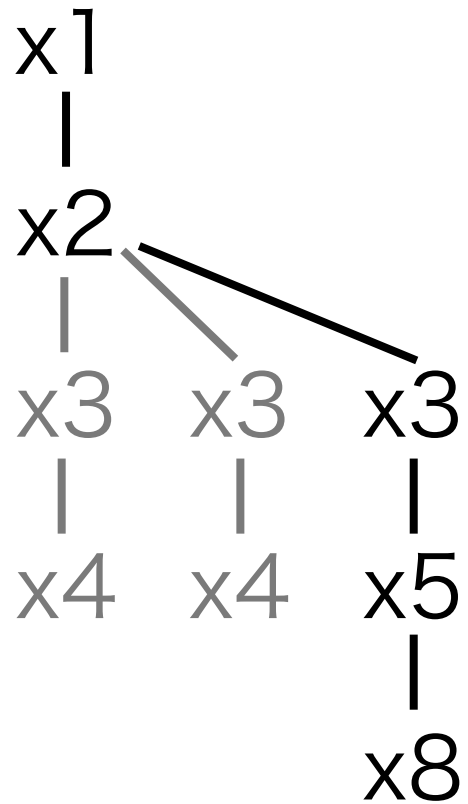
Implication Graph



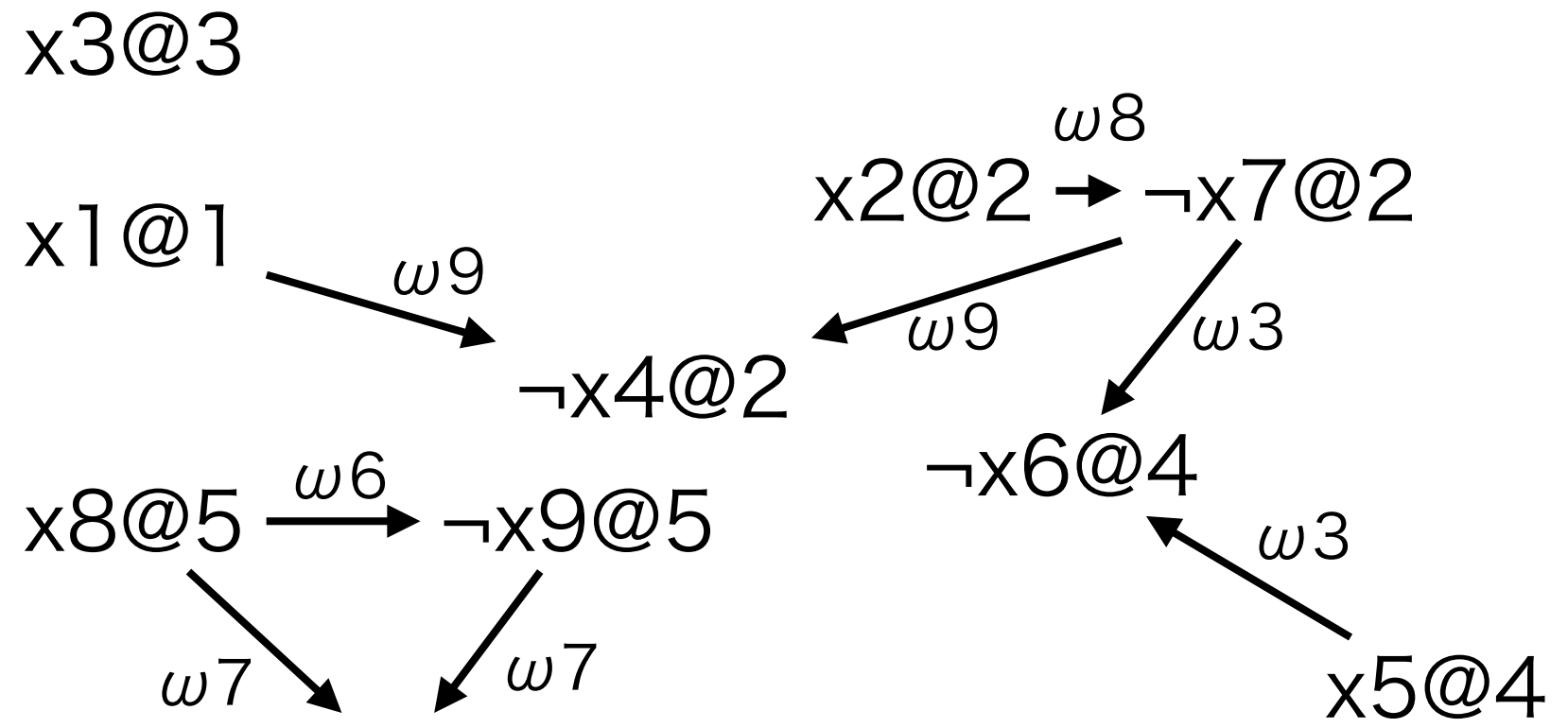
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Deduce



Implication Graph



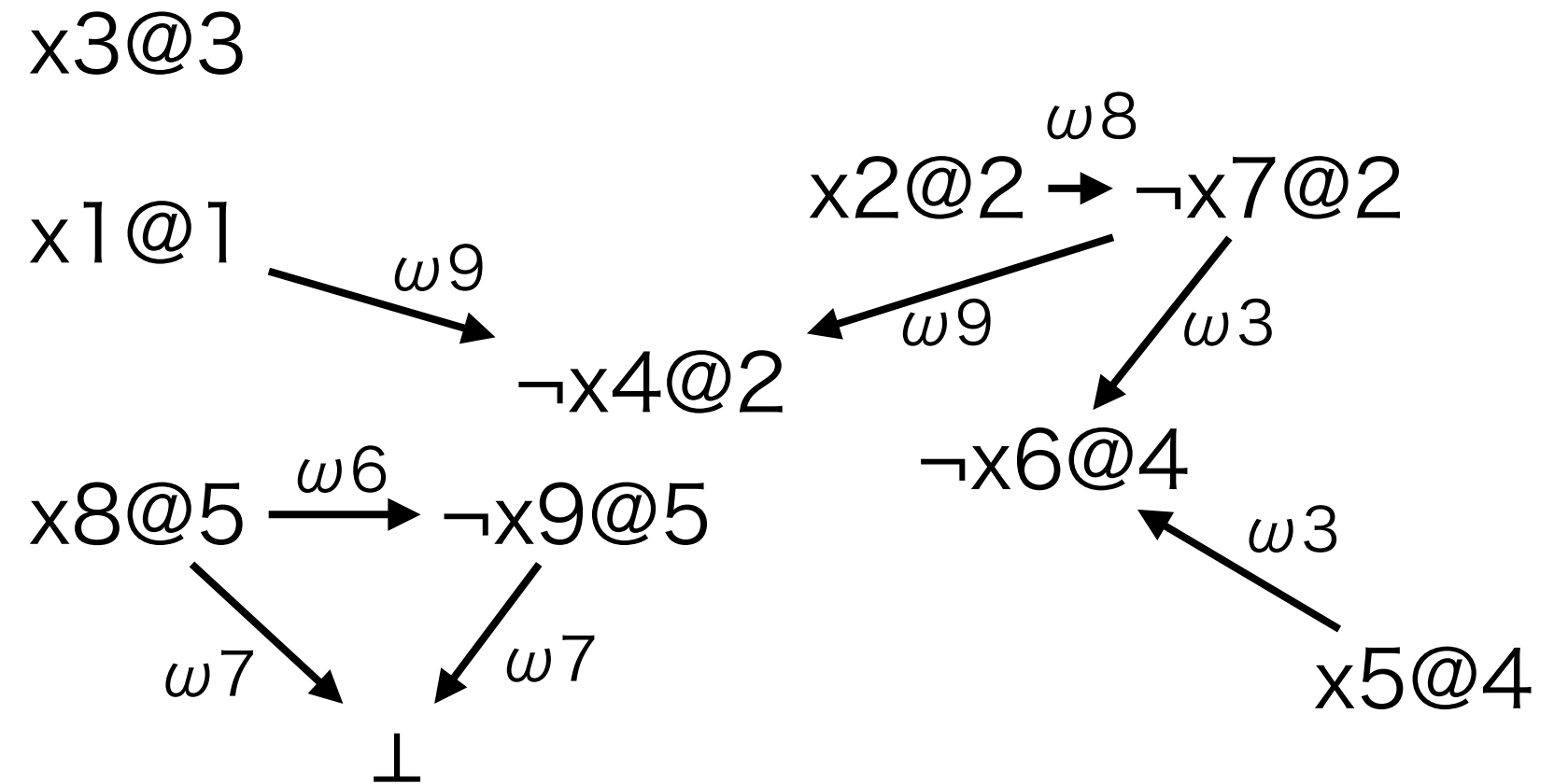
Clause DB

Conflict

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Diagnose

Implication Graph



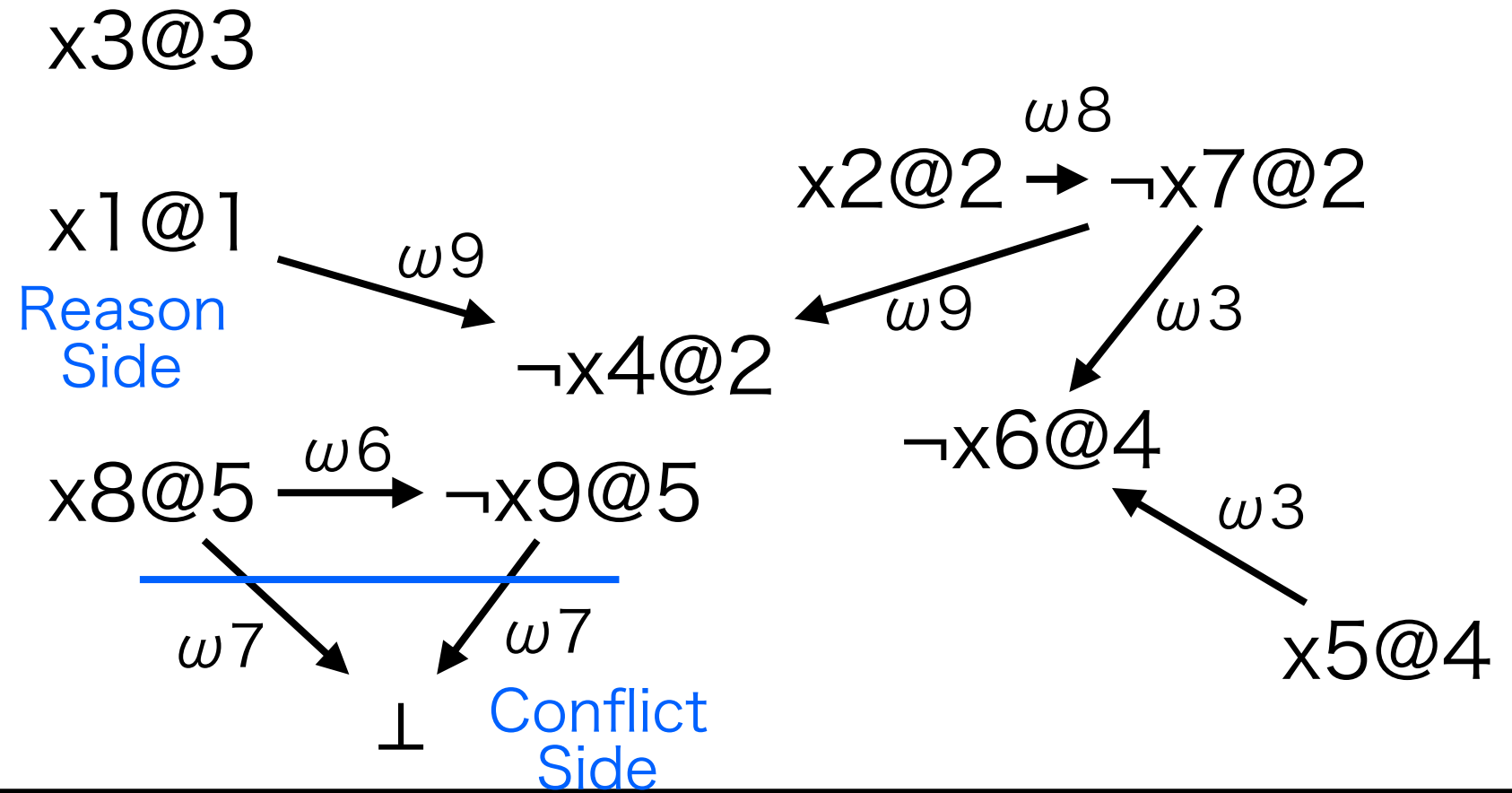
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Diagnose

Implication Graph

$\omega 7:$
 $\neg x8 \vee x9$



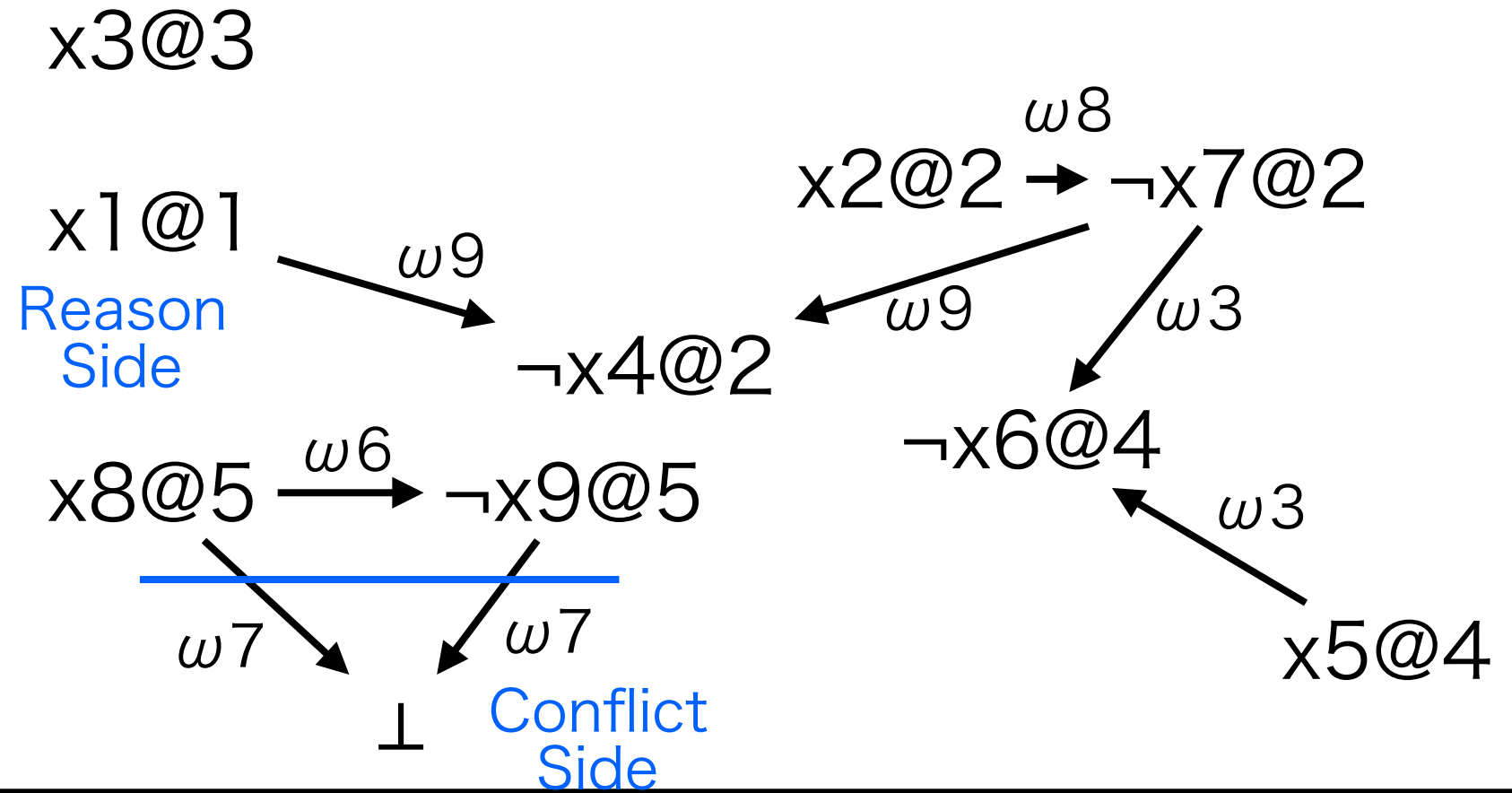
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Diagnose

$\omega 7: \neg x8 \vee x9$ $\omega 6: \neg x8 \vee \neg x9$

Implication Graph



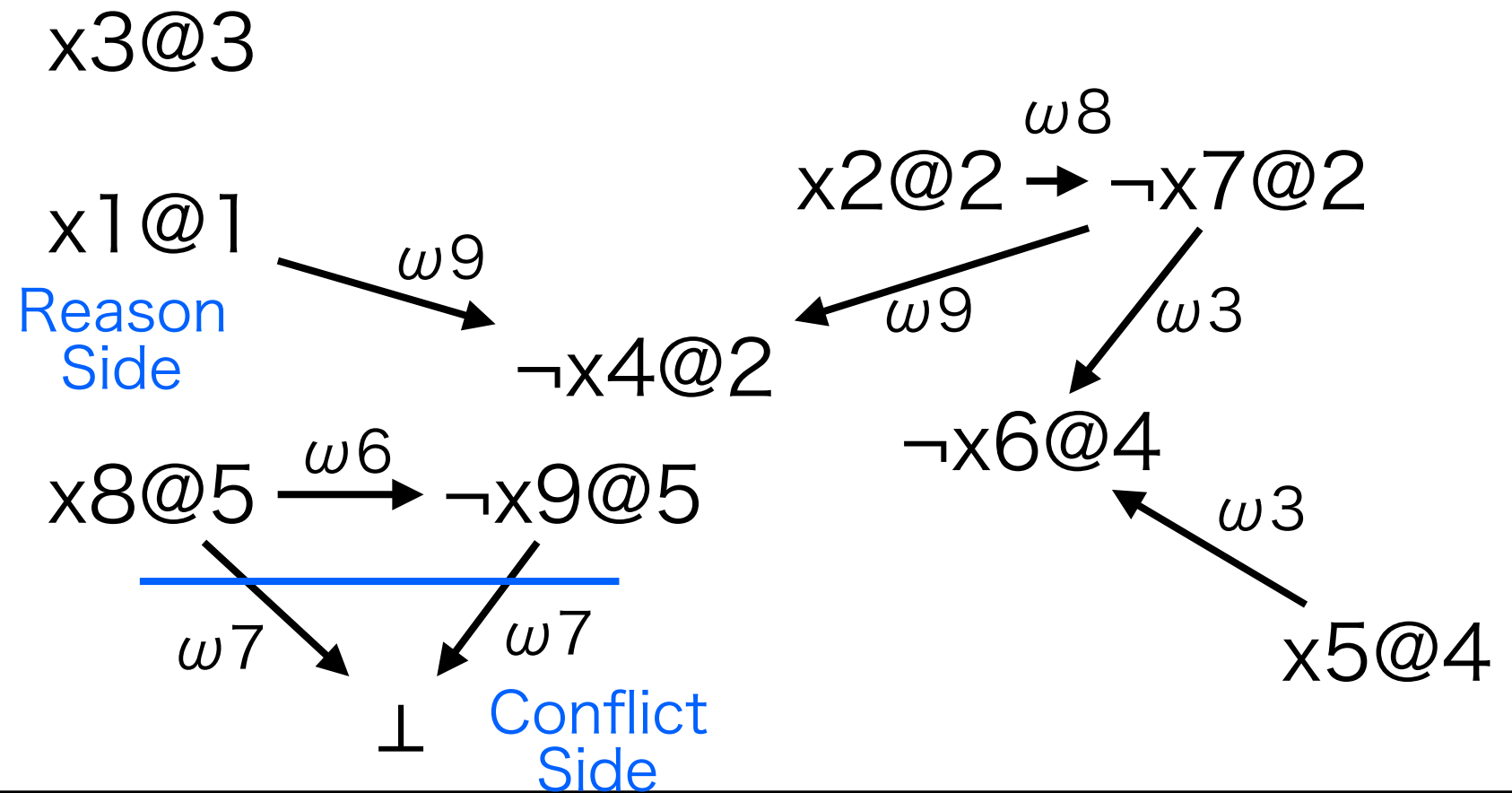
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Diagnose

$\omega 7: \neg x8 \vee x9$ $\omega 6: \neg x8 \vee \neg x9$

Implication Graph



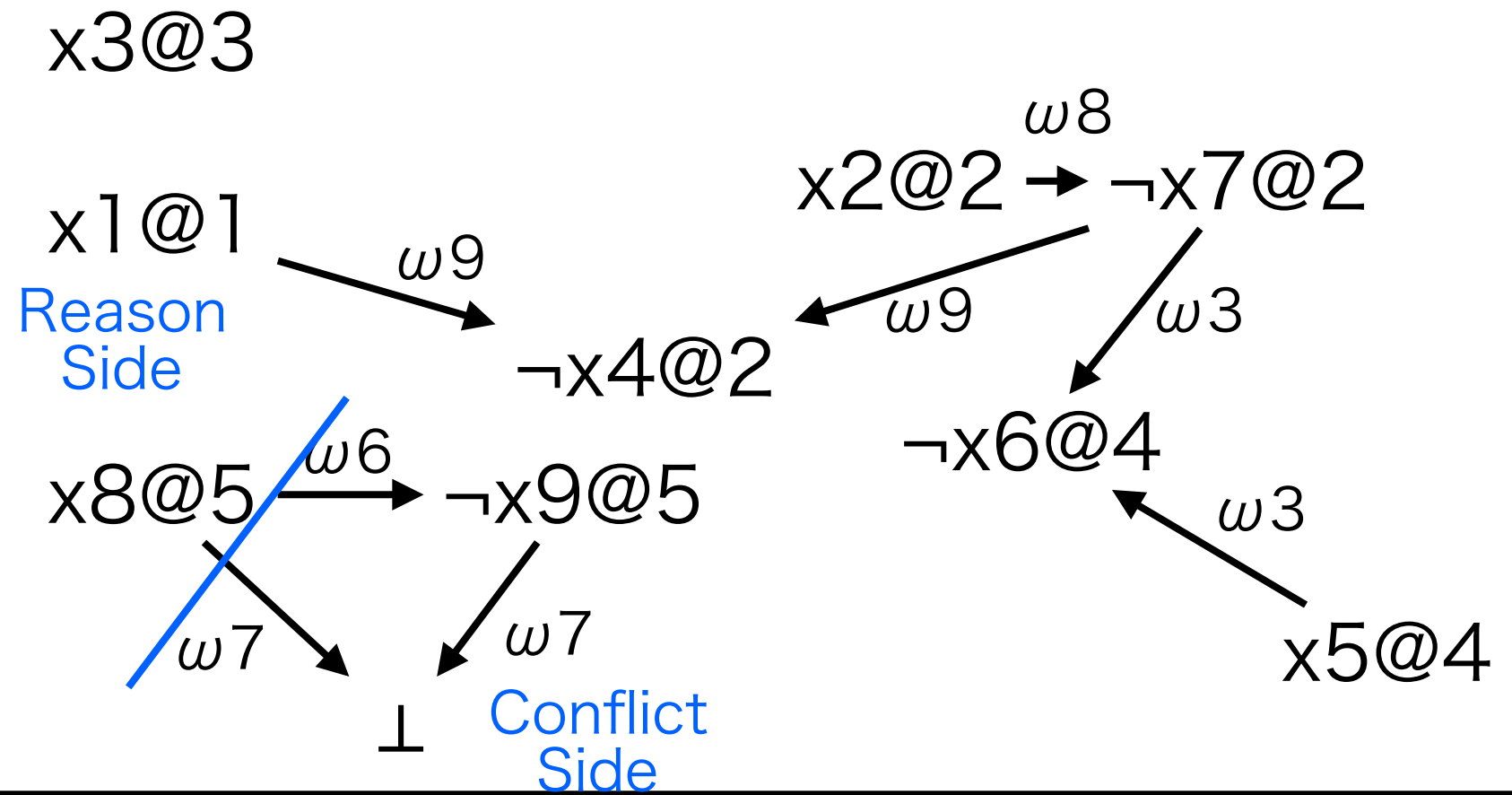
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Diagnose

$$\frac{\omega 7: \neg x8 \vee x9 \quad \omega 6: \neg x8 \vee \neg x9}{\neg x8}$$

Implication Graph



Clause DB

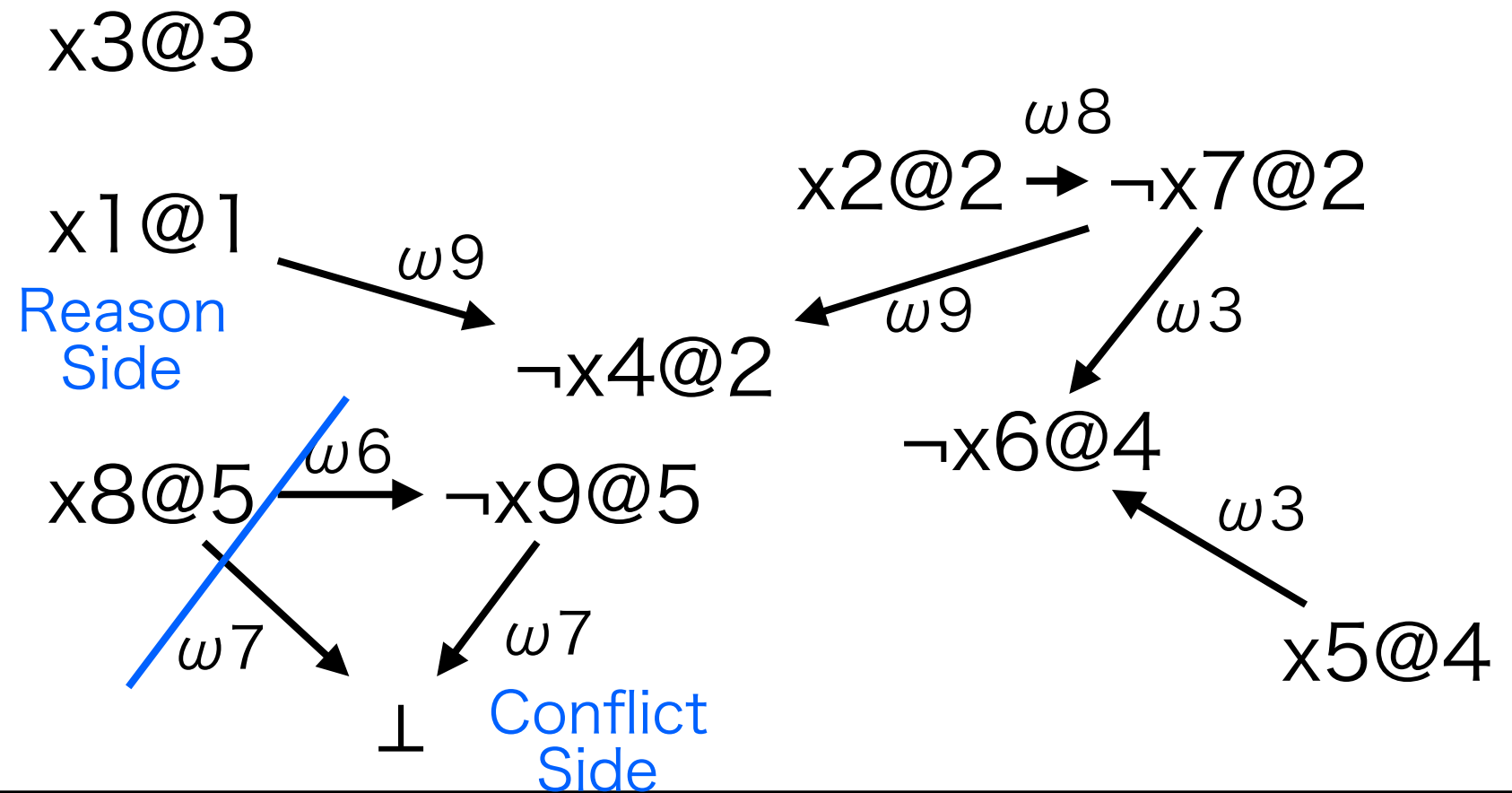
- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Diagnose

$$\frac{\omega 7: \neg x8 \vee x9 \quad \omega 6: \neg x8 \vee \neg x9}{\neg x8}$$

It contains one literal
in the current
decision level (i.e. 5).

Implication Graph



Clause DB

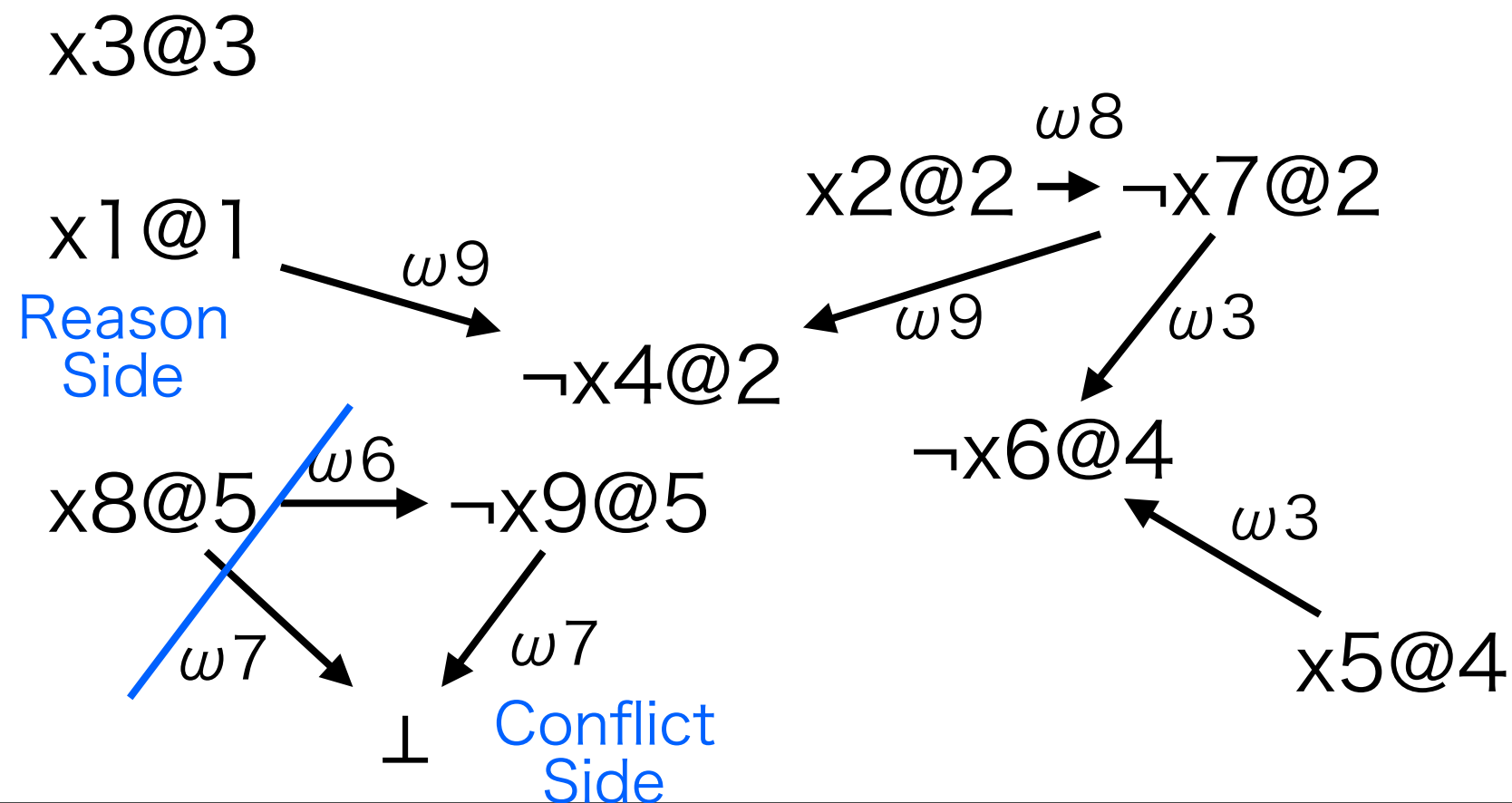
- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$

Diagnose

$$\frac{\omega 7: \neg x8 \vee x9 \quad \omega 6: \neg x8 \vee \neg x9}{\neg x8}$$

It contains one literal
in the current
decision level (i.e. 5).

Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$

- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$

New clause is learnt !

- $\omega 10: \neg x8$

Backtrack

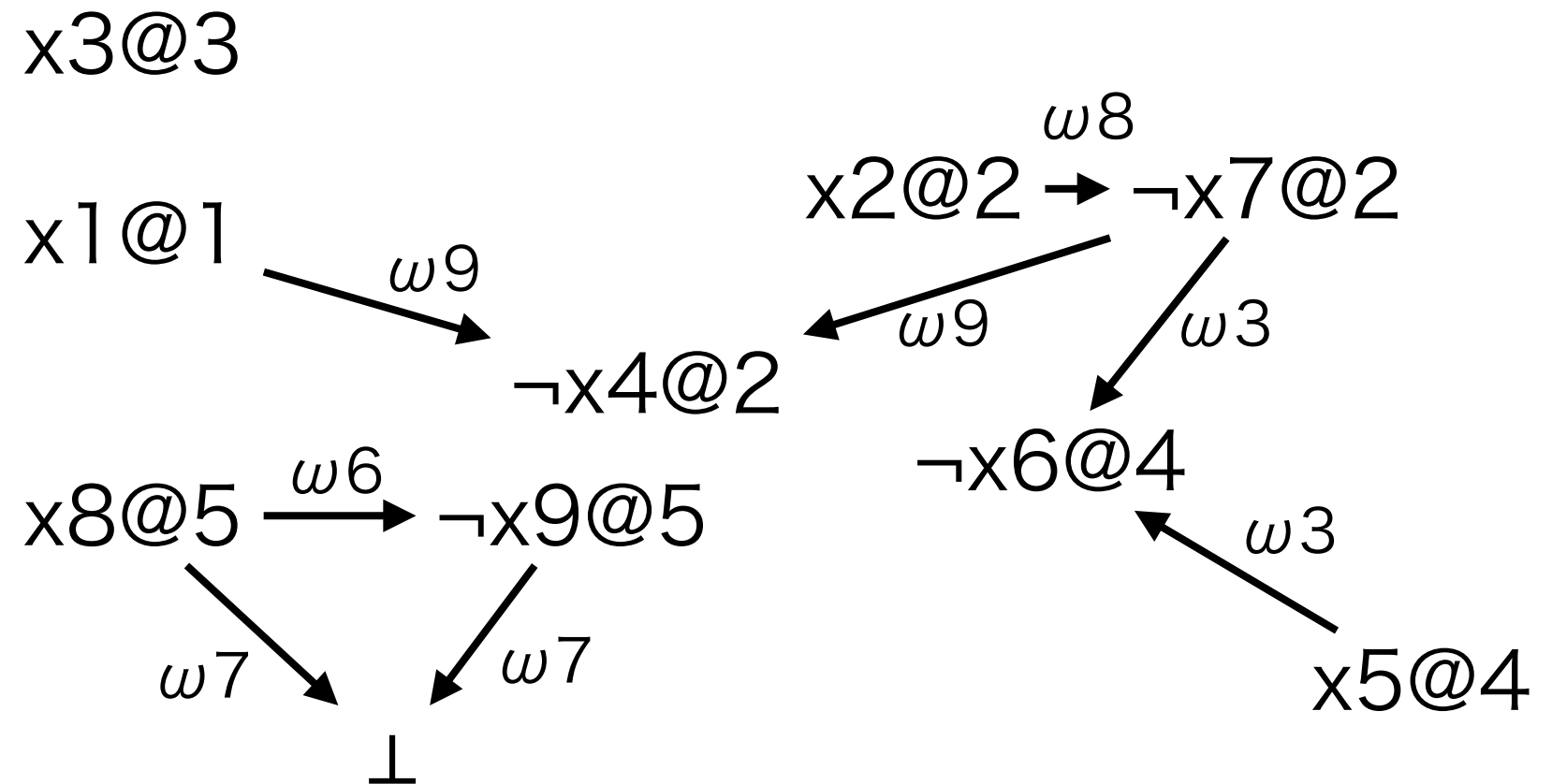
Learnt clause:
 $\neg x_8$

x_8 was set at level 5

Assertion level of this clause (which is the second highest level) is 0 by convention.

$\Rightarrow$ Backtrack to level 0 (root level)

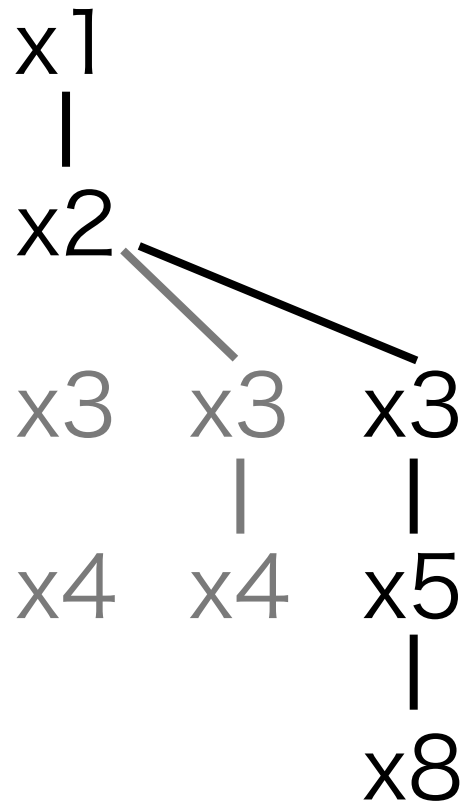
Implication Graph



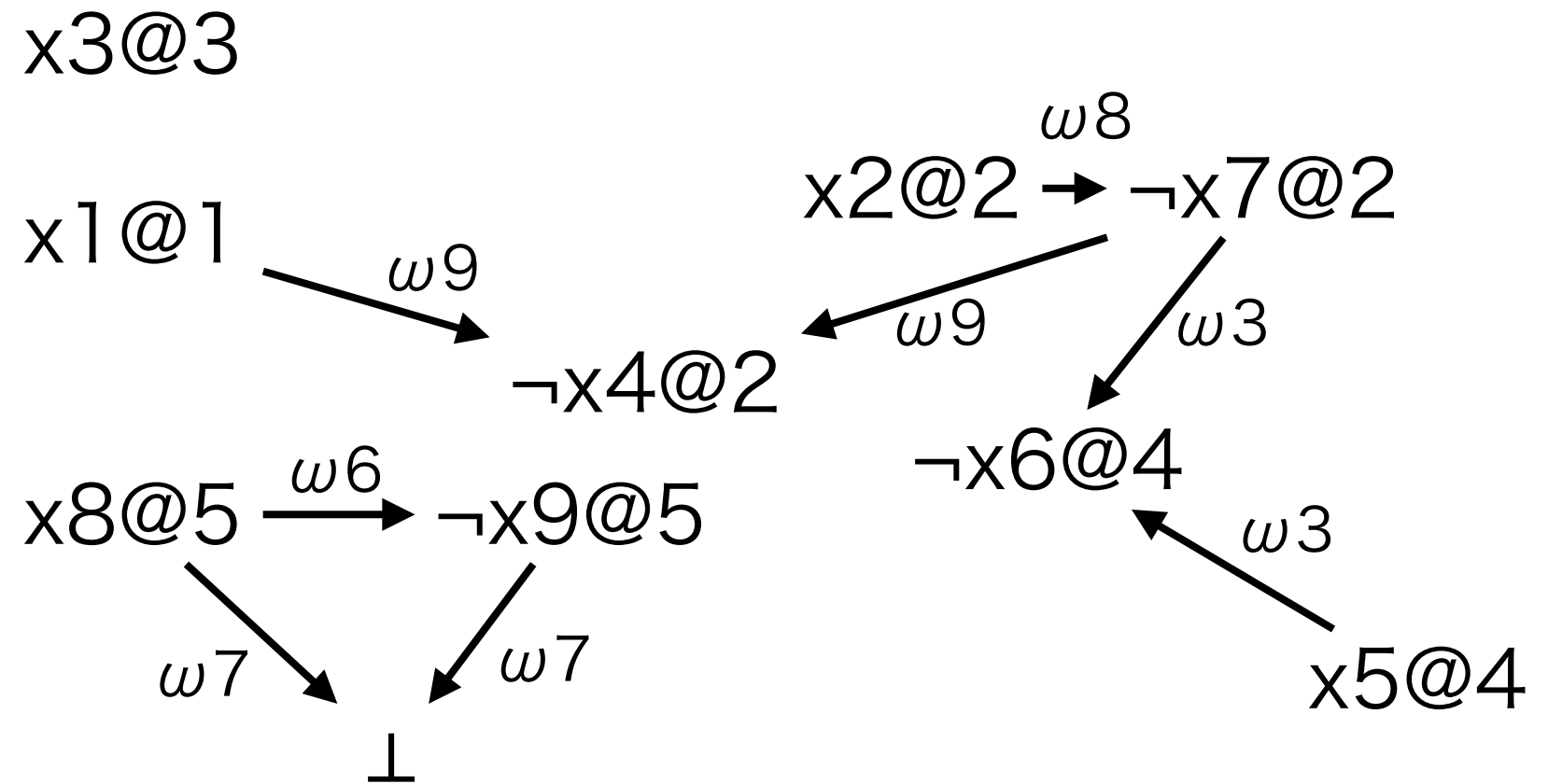
Clause DB

- $\omega 1: \neg x_1 \vee \neg x_4 \vee x_5$
- $\omega 2: \neg x_4 \vee x_6$
- $\omega 3: \neg x_5 \vee \neg x_6 \vee x_7$
- $\omega 4: \neg x_7 \vee x_8$
- $\omega 5: \neg x_2 \vee \neg x_7 \vee x_9$
- $\omega 6: \neg x_8 \vee \neg x_9$
- $\omega 7: \neg x_8 \vee x_9$
- $\omega 8: \neg x_2 \vee \neg x_7$
- $\omega 9: \neg x_1 \vee \neg x_4 \vee x_7$
- $\omega 10: \neg x_8$

Backtrack



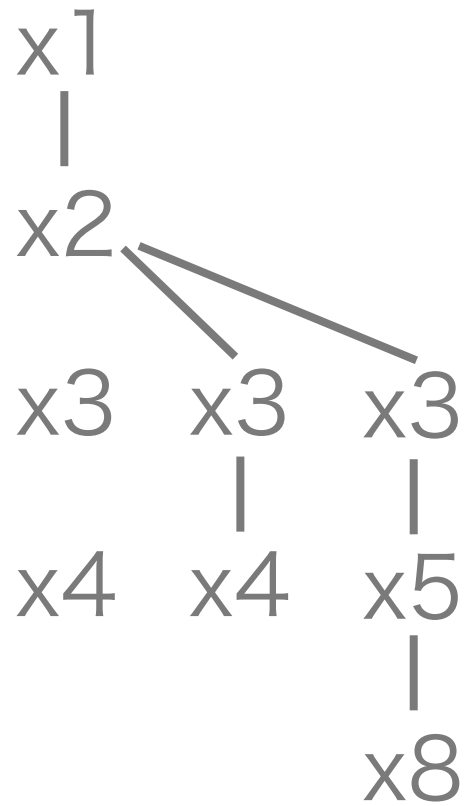
Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Backtrack

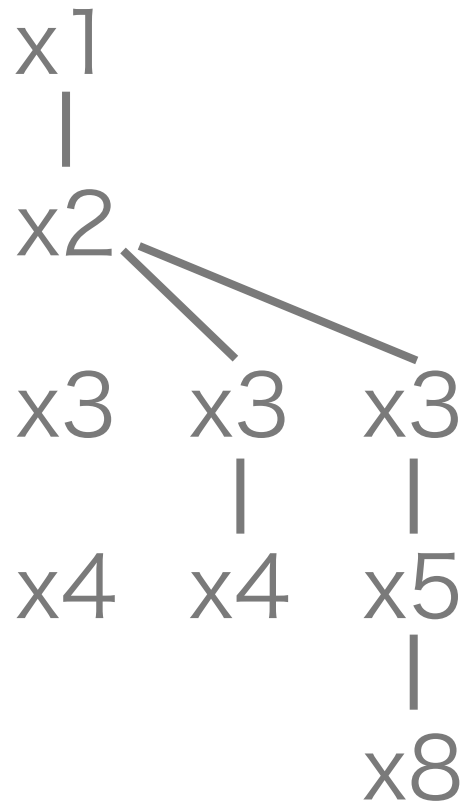


Implication Graph

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Deduce

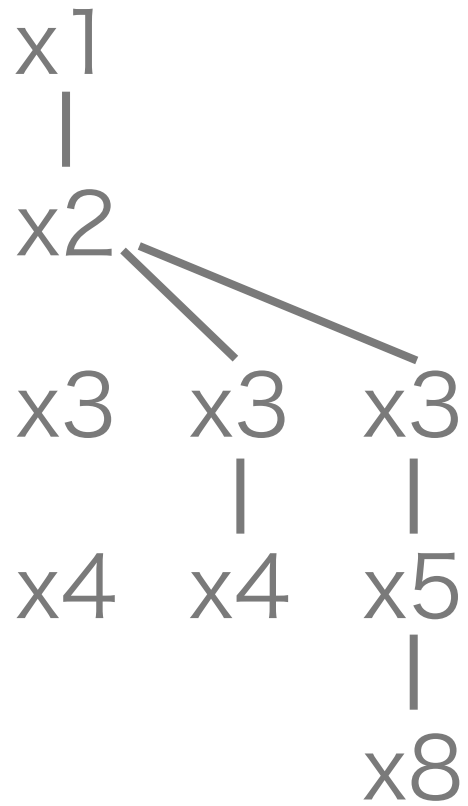


Implication Graph

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
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- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Deduce



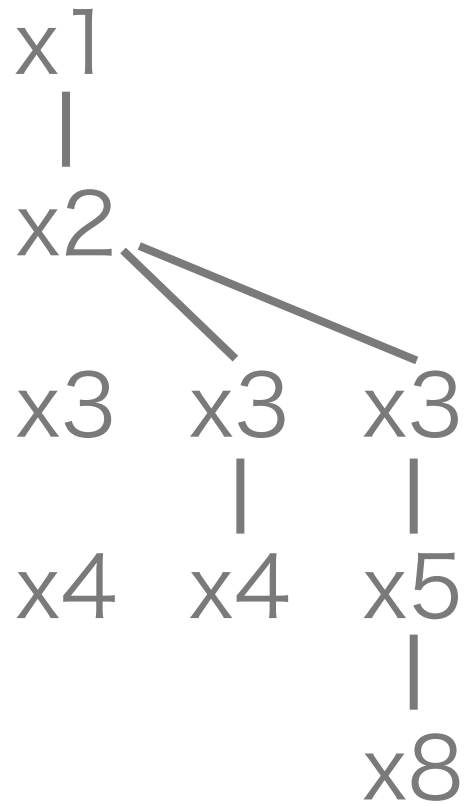
$\neg x8@0$

Implication Graph

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Decide



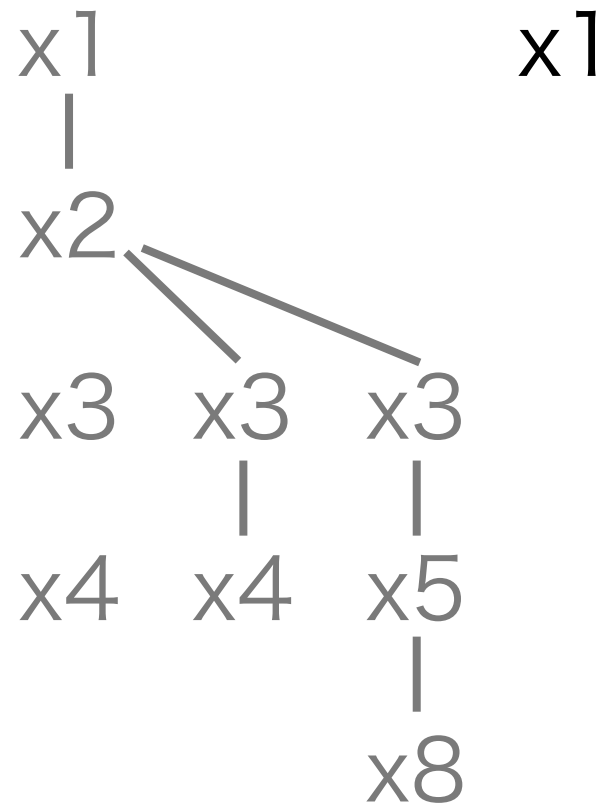
Implication Graph

$\neg x8@0$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Decide



Implication Graph

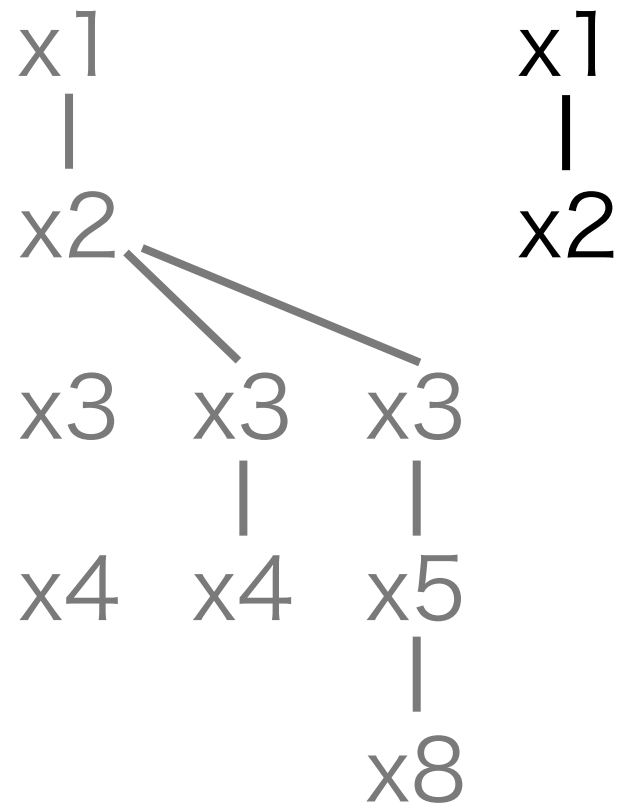
$x1@1$

$\neg x8@0$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Decide



Implication Graph

$x1@1$

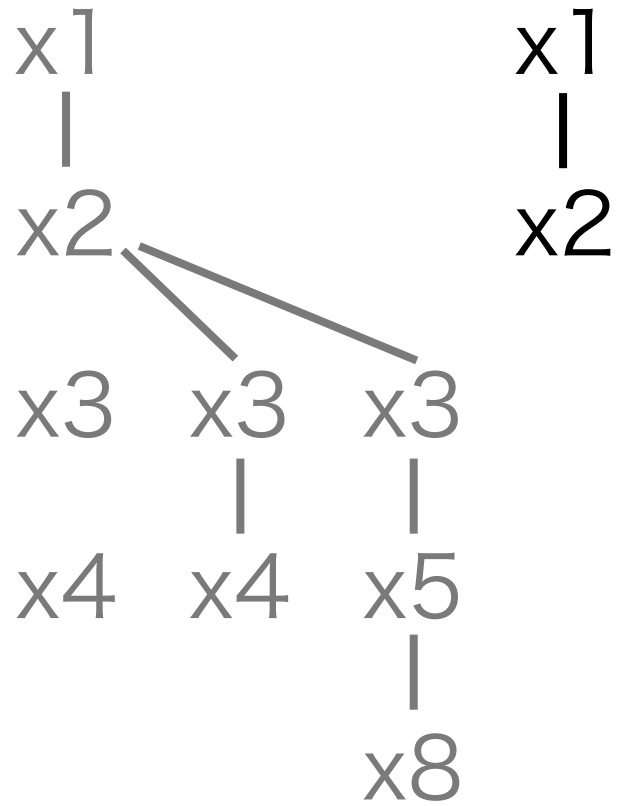
$x2@2$

$\neg x8@0$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Deduce



Implication Graph

$x1@1$

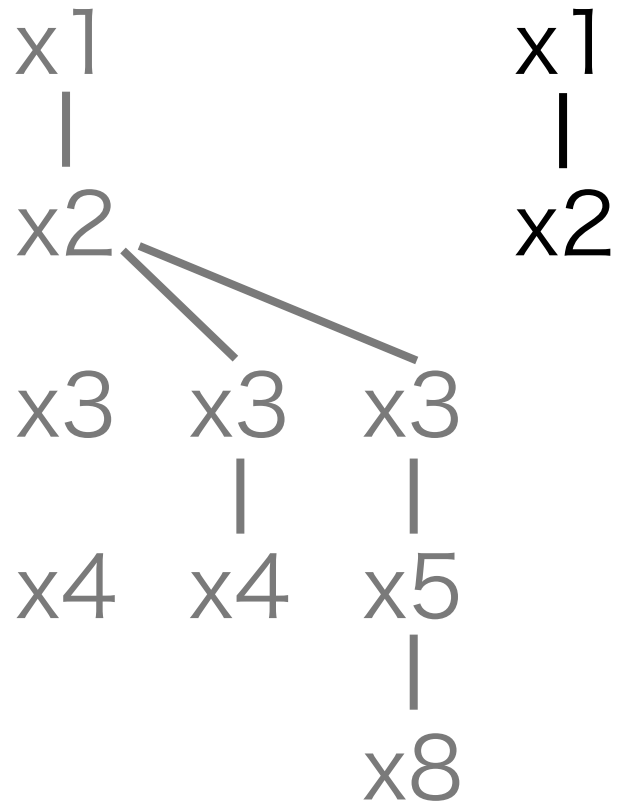
$x2@2$

$\neg x8@0$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Deduce



Implication Graph

$x1@1$

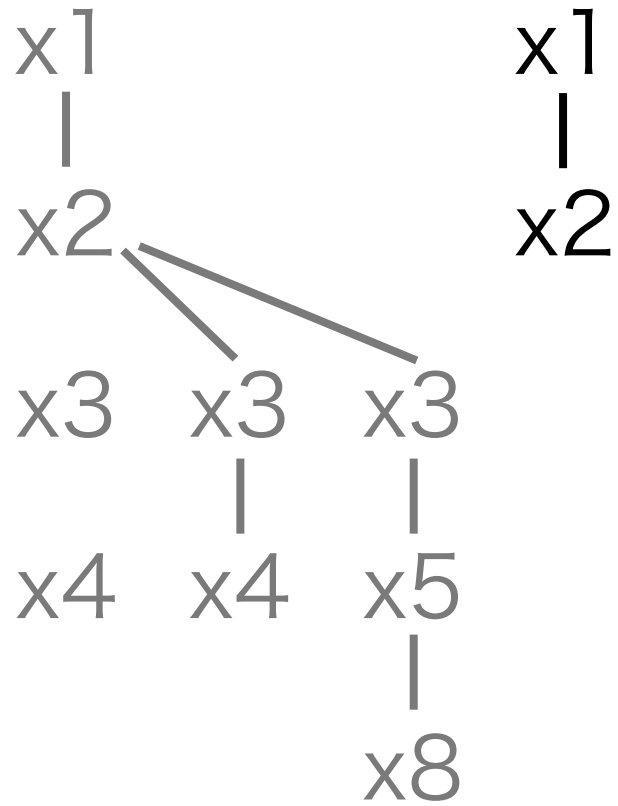
$\neg x8@0$

$x2@2 \xrightarrow{\omega 8} \neg x7@2$

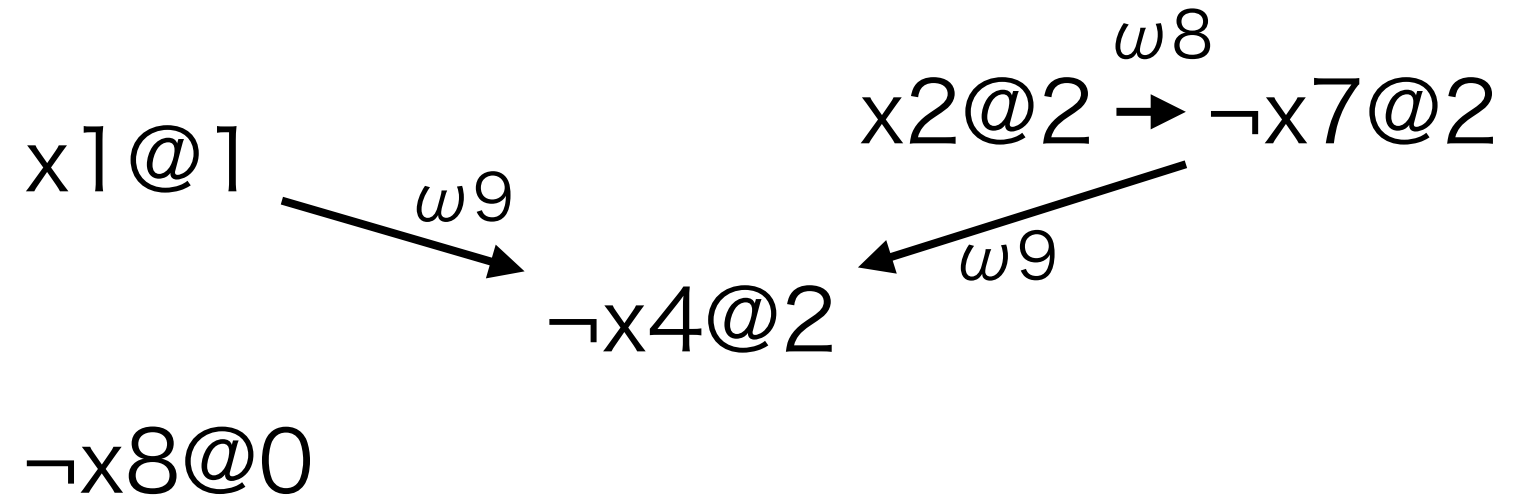
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Deduce



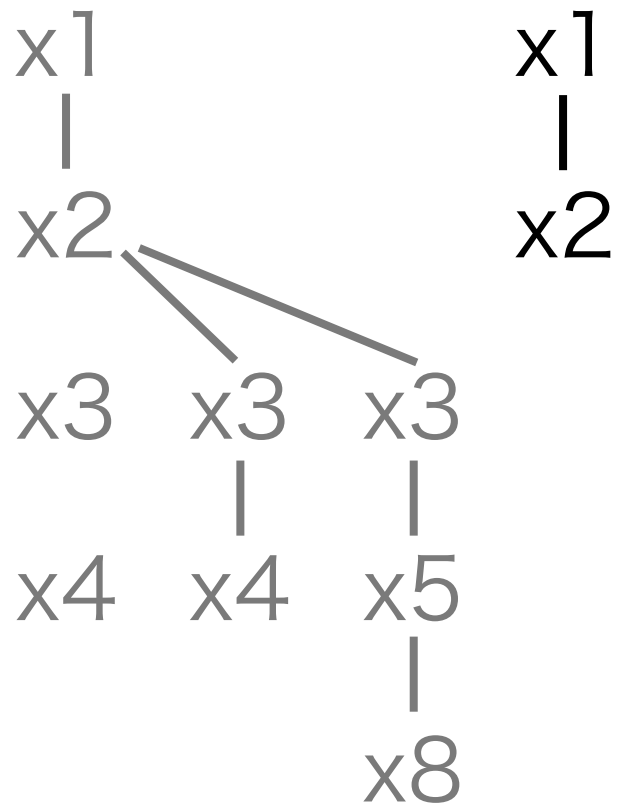
Implication Graph



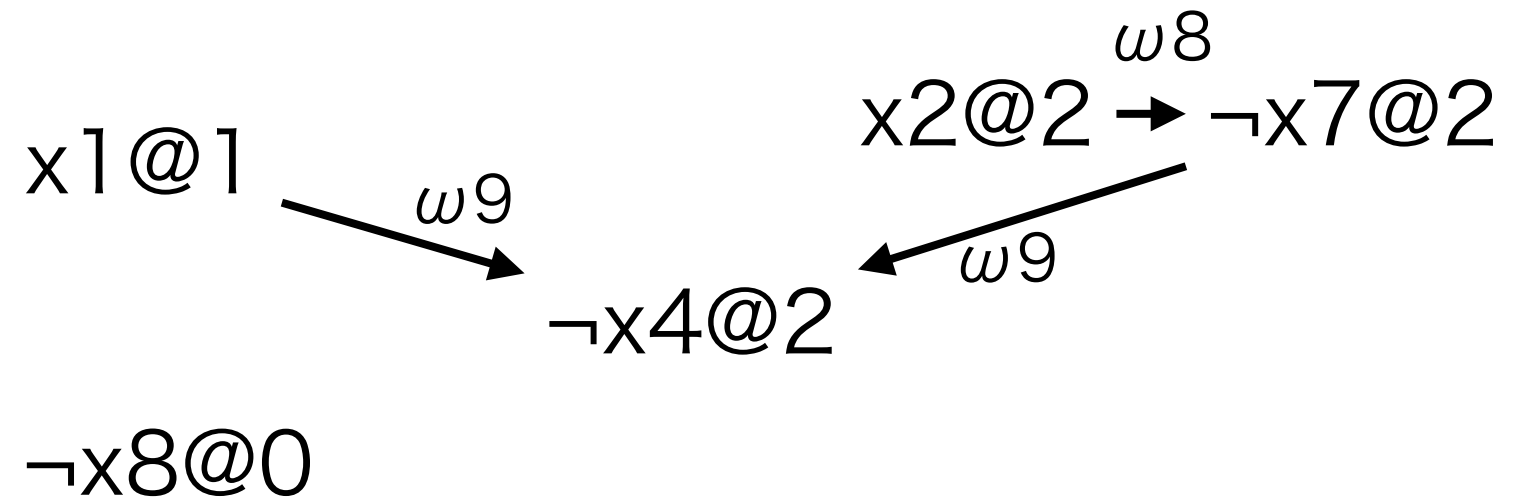
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Decide



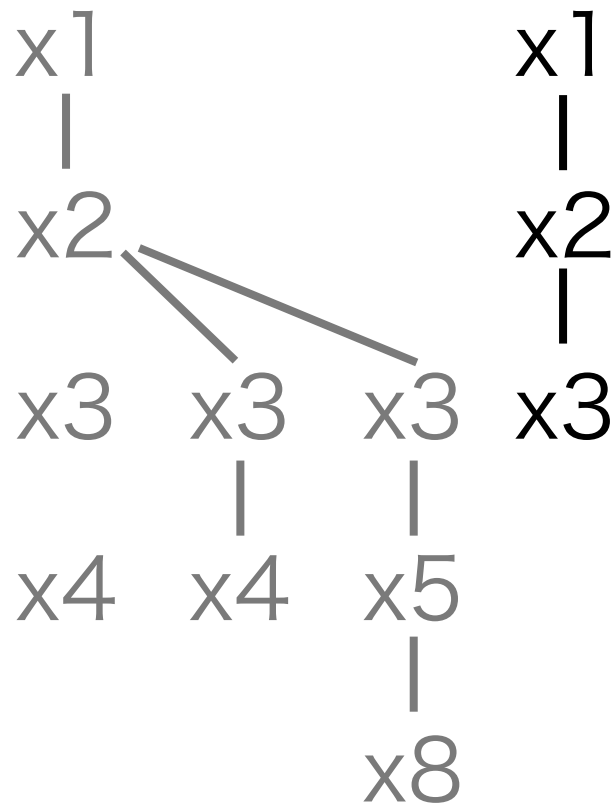
Implication Graph



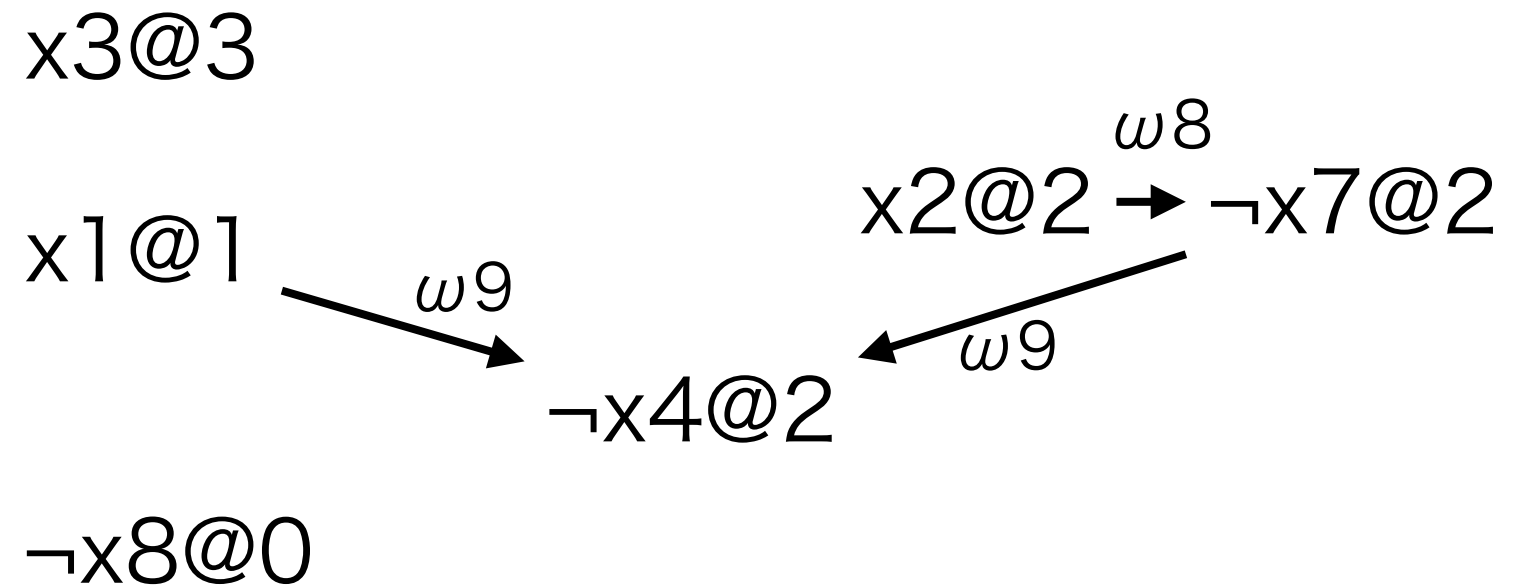
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Decide



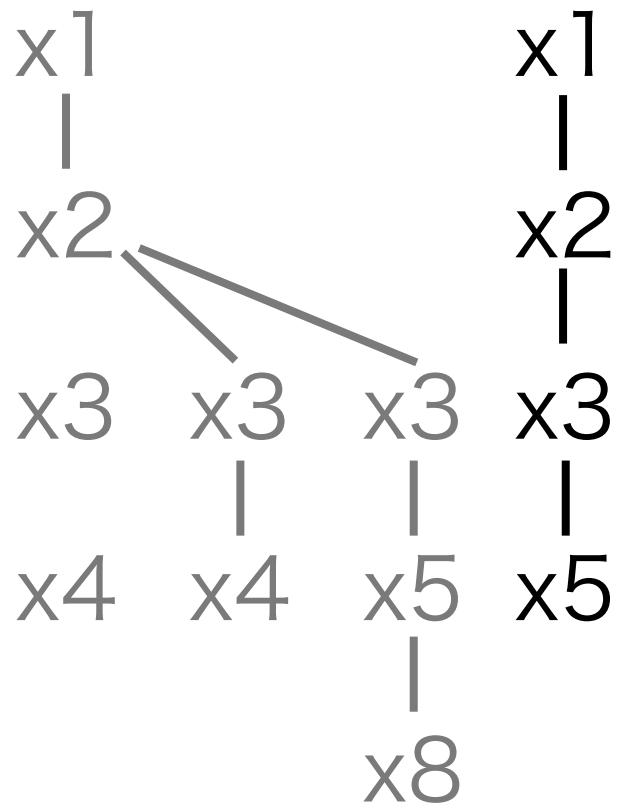
Implication Graph



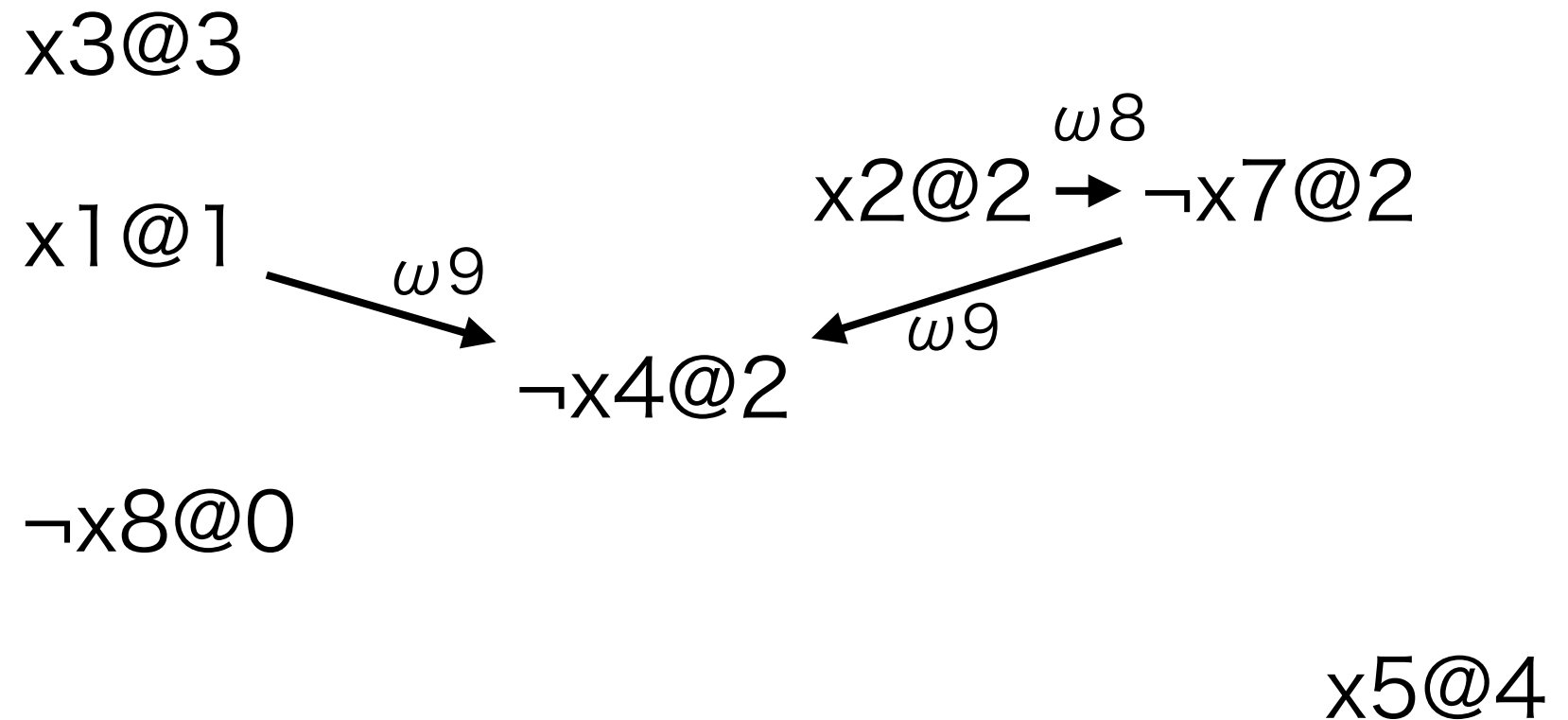
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Decide



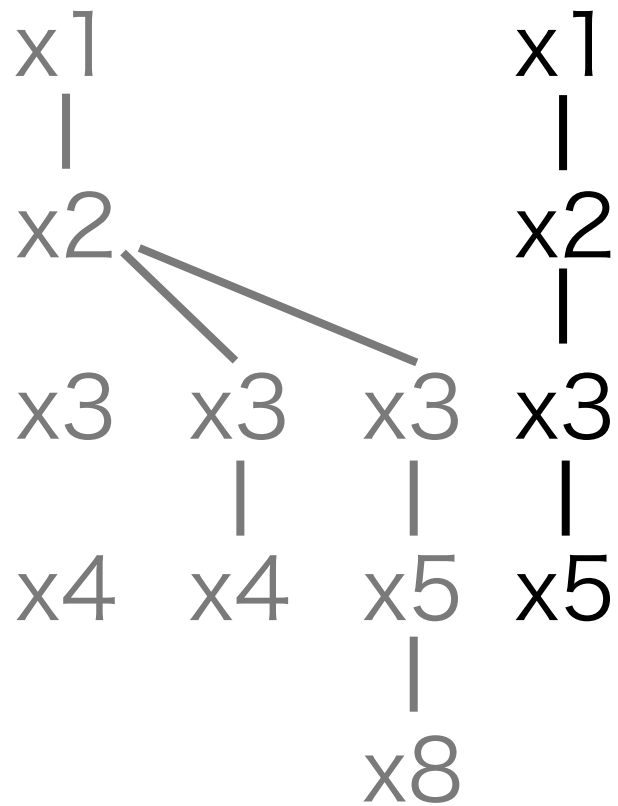
Implication Graph



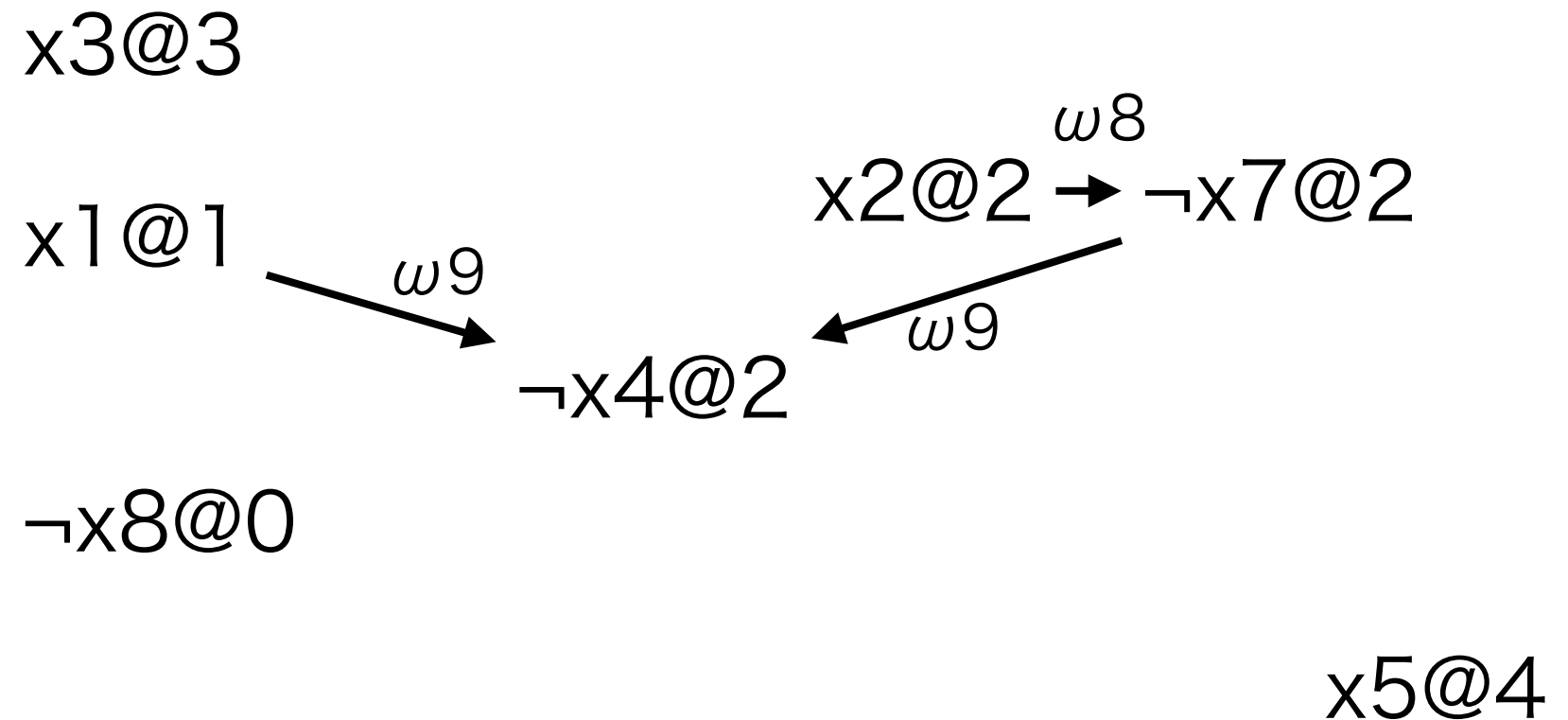
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Deduce



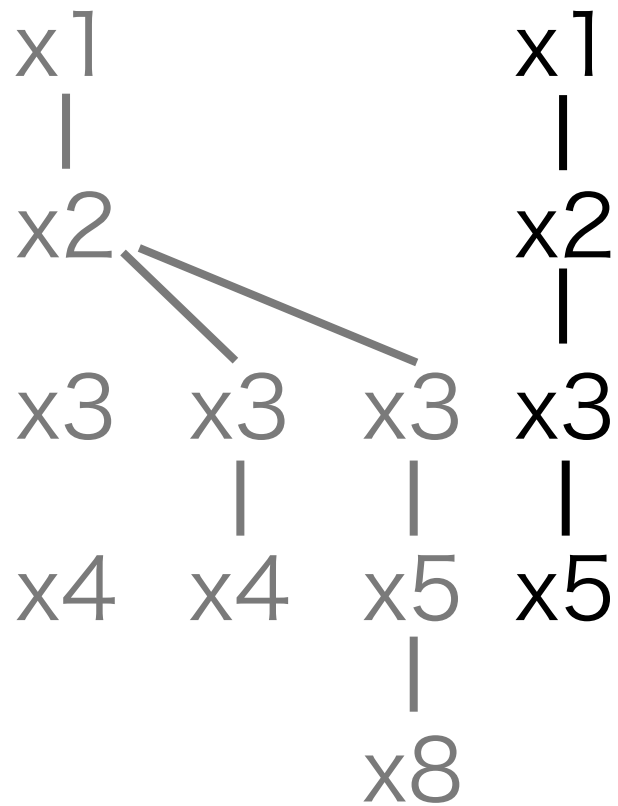
Implication Graph



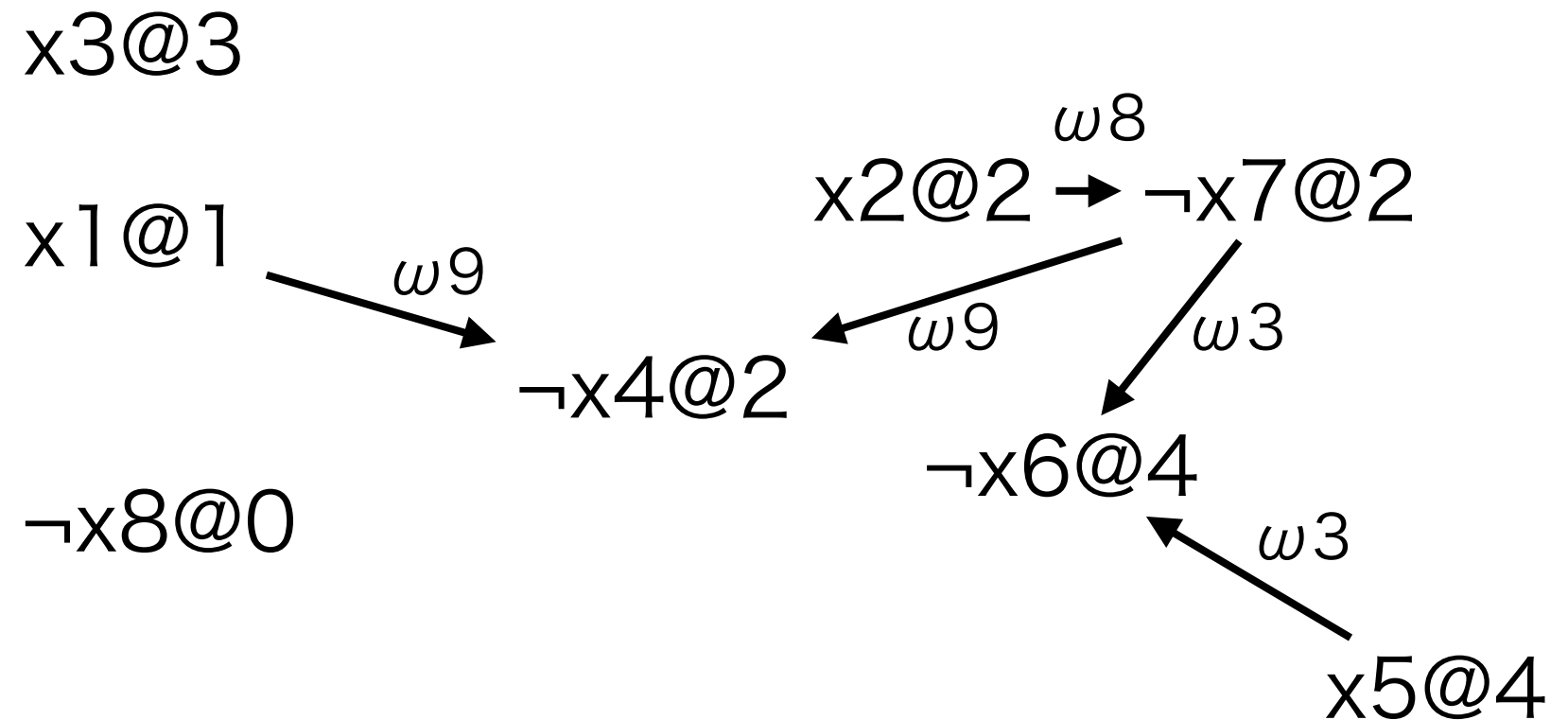
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Deduce



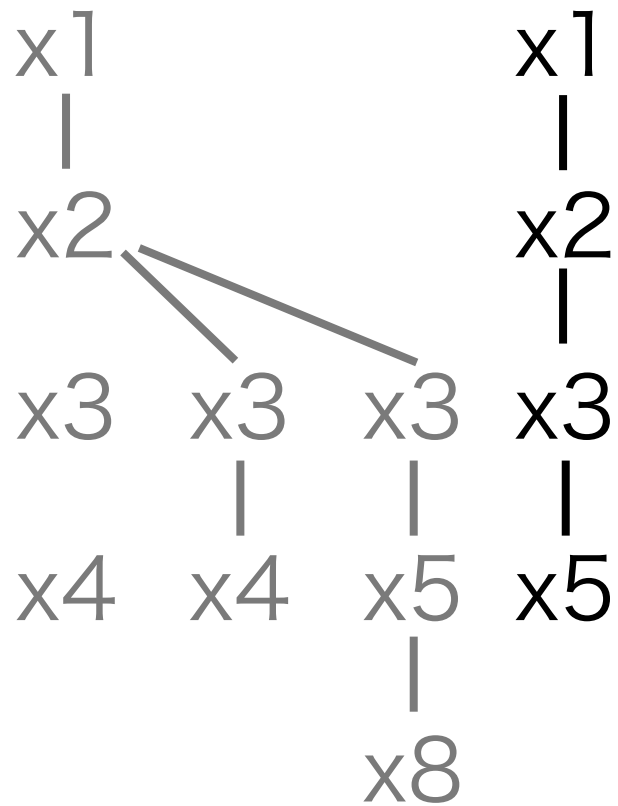
Implication Graph



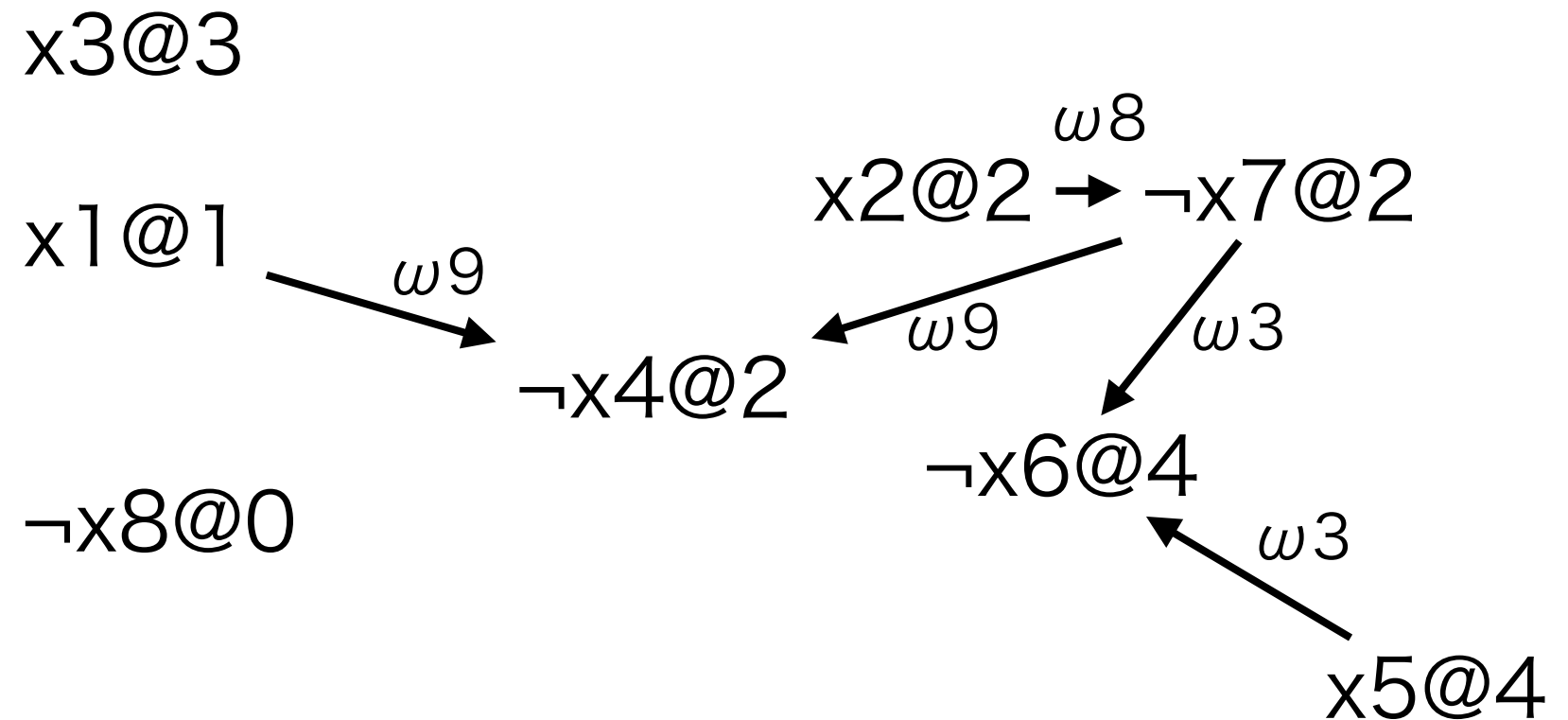
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Decide



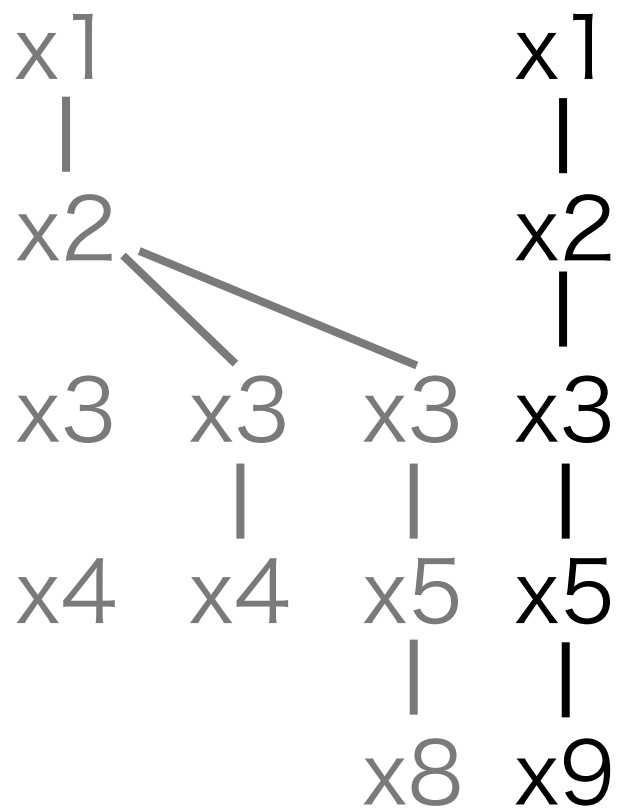
Implication Graph



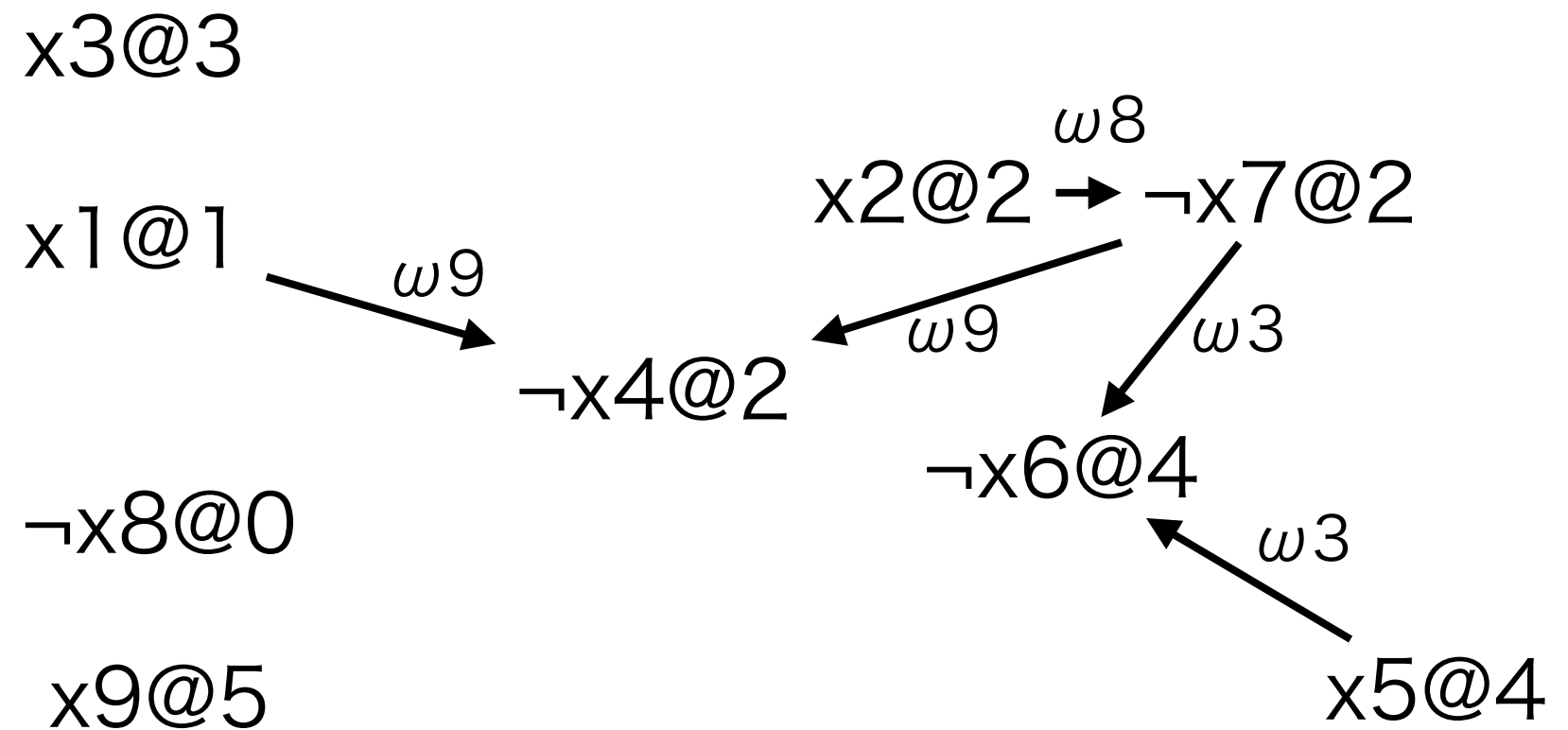
Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
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- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Decide



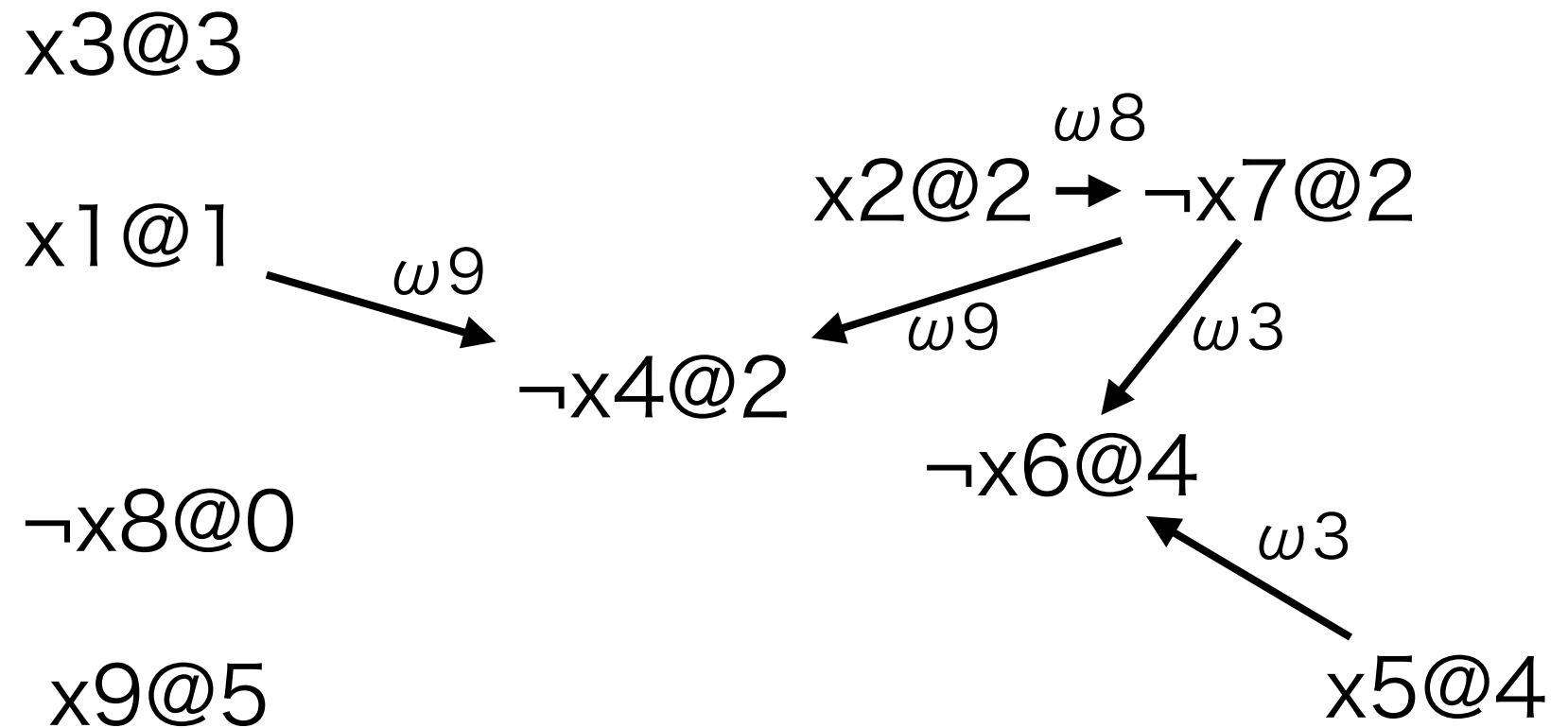
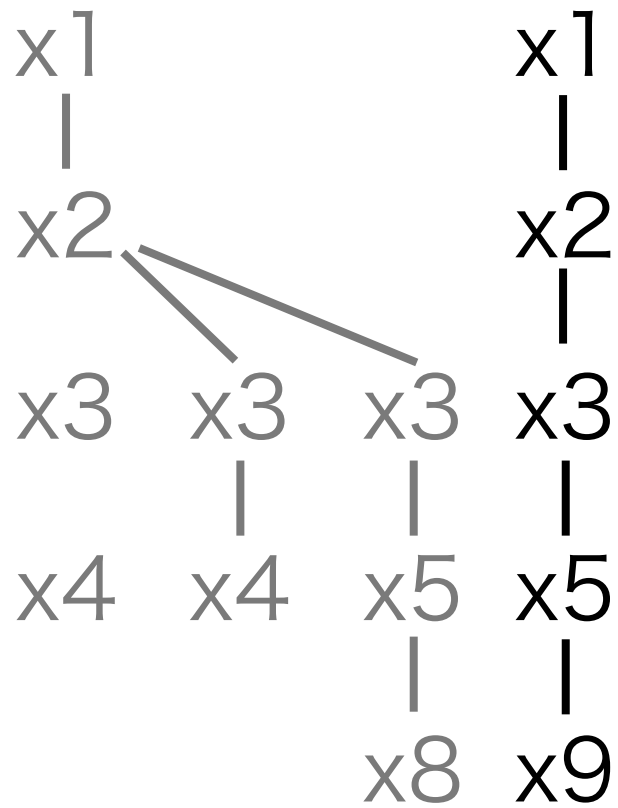
Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

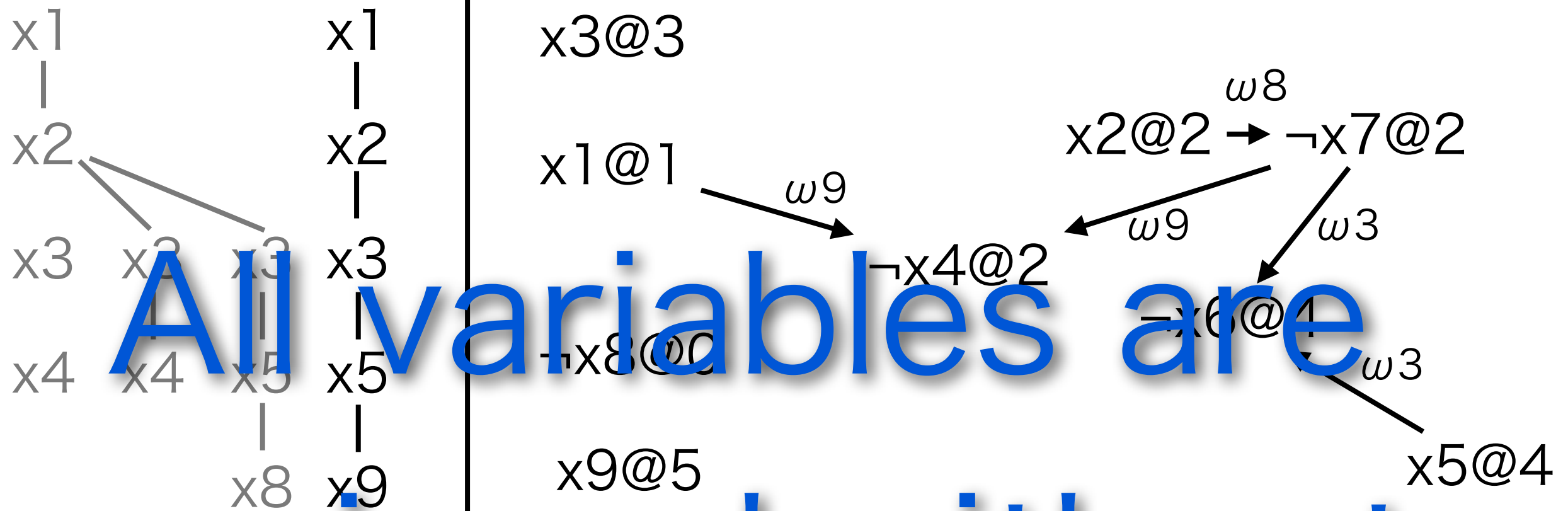
Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

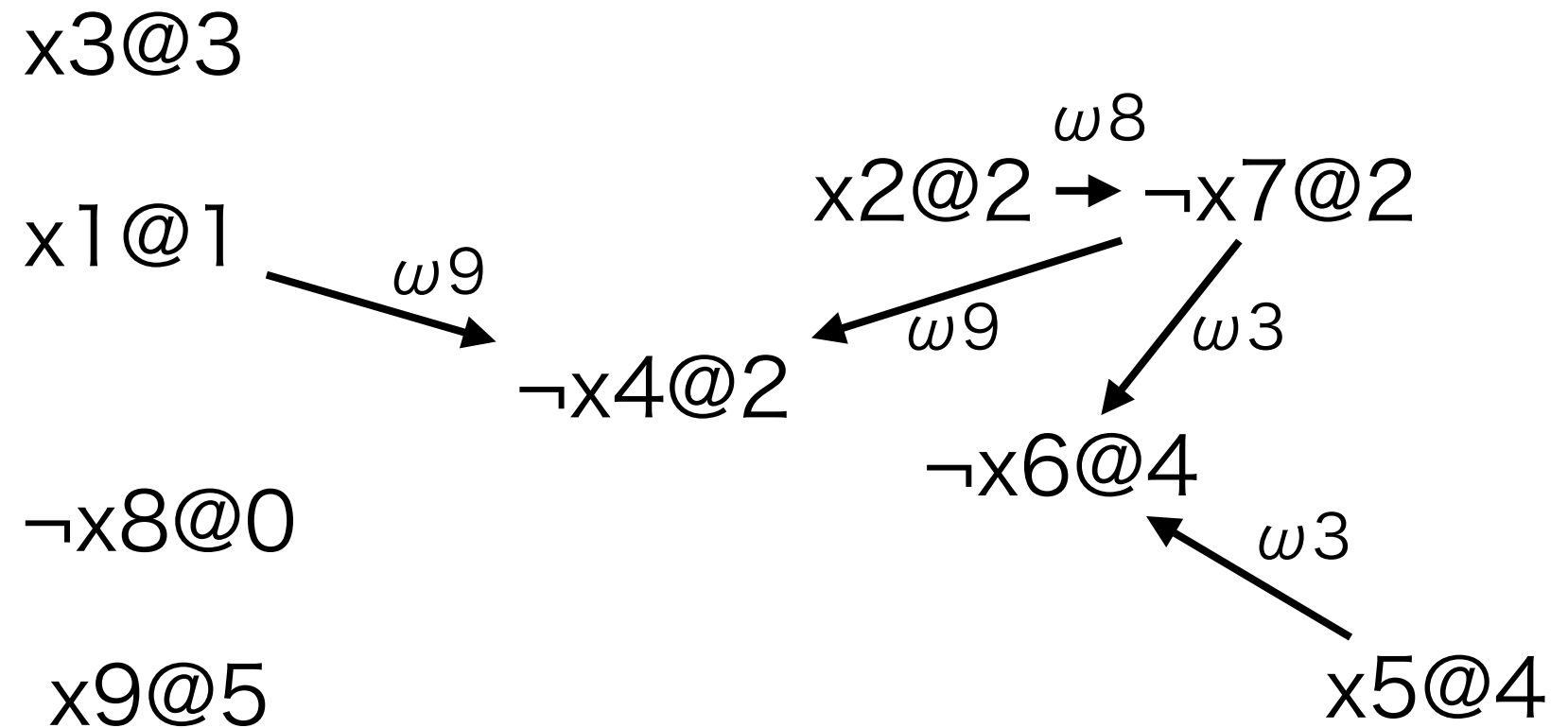
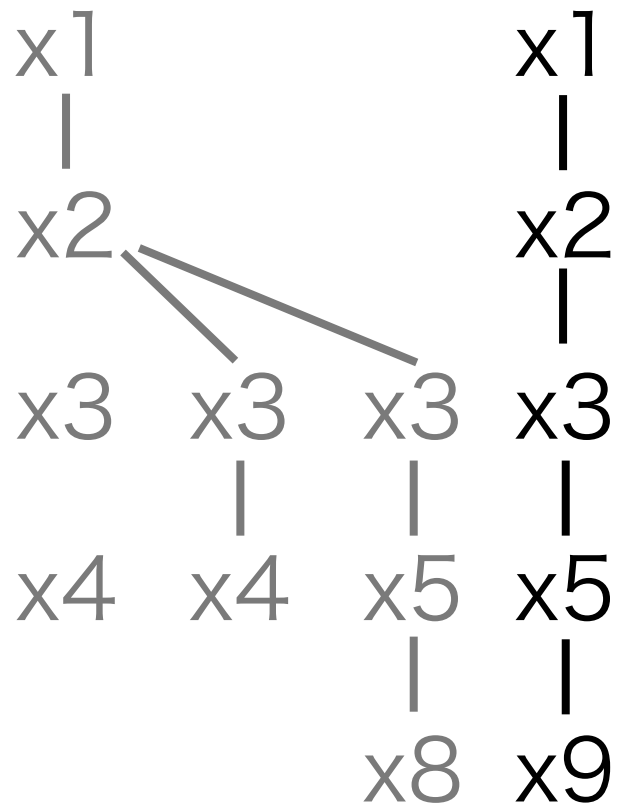
Implication Graph



All variables are assigned without conflict

- $w1: \neg x1 \vee \neg x4 \vee x5$
- $w2: \neg x4 \vee x6$
- $w3: \neg x5 \vee \neg x6 \vee x7$
- $w4: \neg x7 \vee x8$
- $w5: \neg x2 \vee \neg x7 \vee x9$
- $w6: \neg x8 \vee \neg x9$
- $w7: \neg x8 \vee x9$
- $w8: \neg x2 \vee \neg x7$
- $w9: \neg x1 \vee \neg x4 \vee x7$
- $w10: \neg x8$

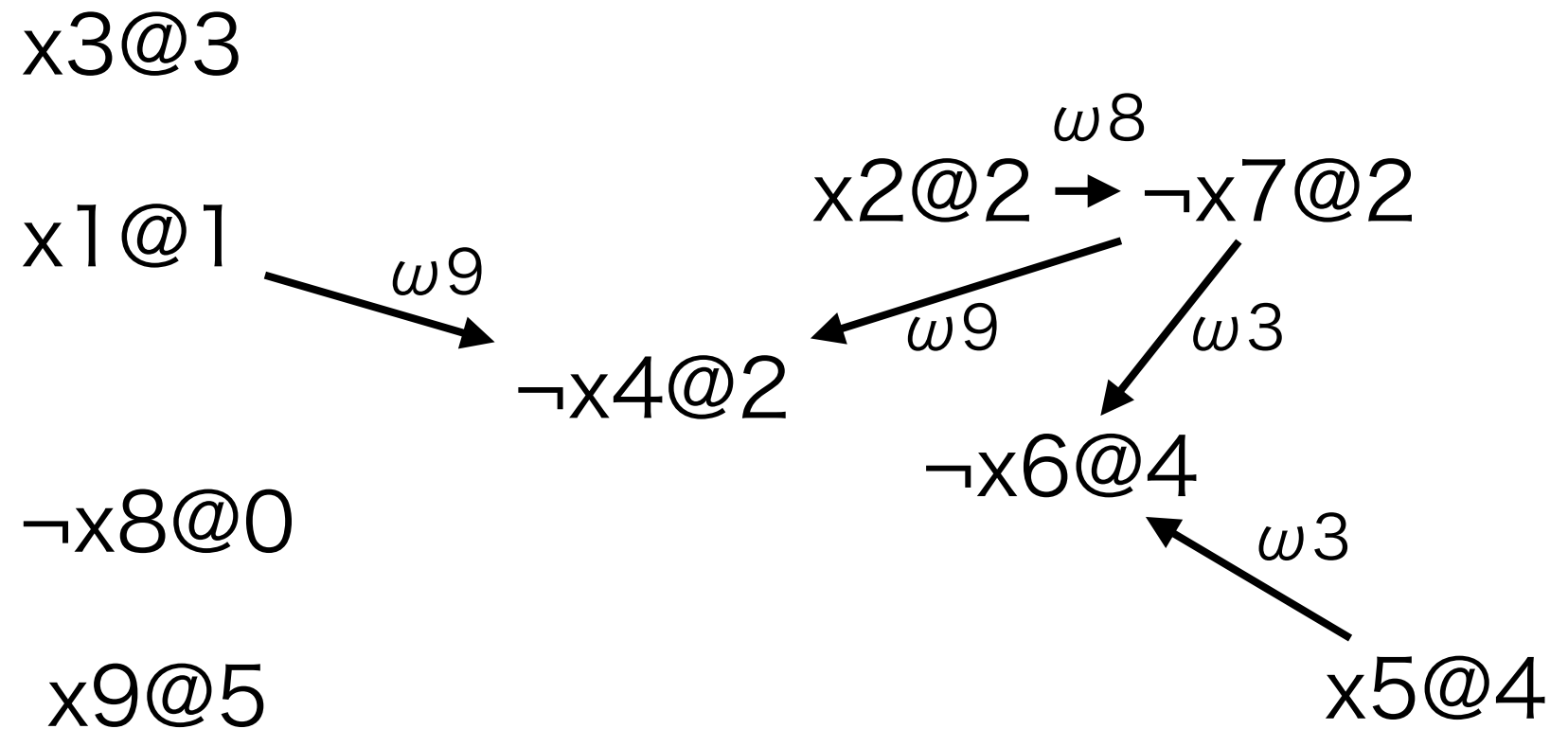
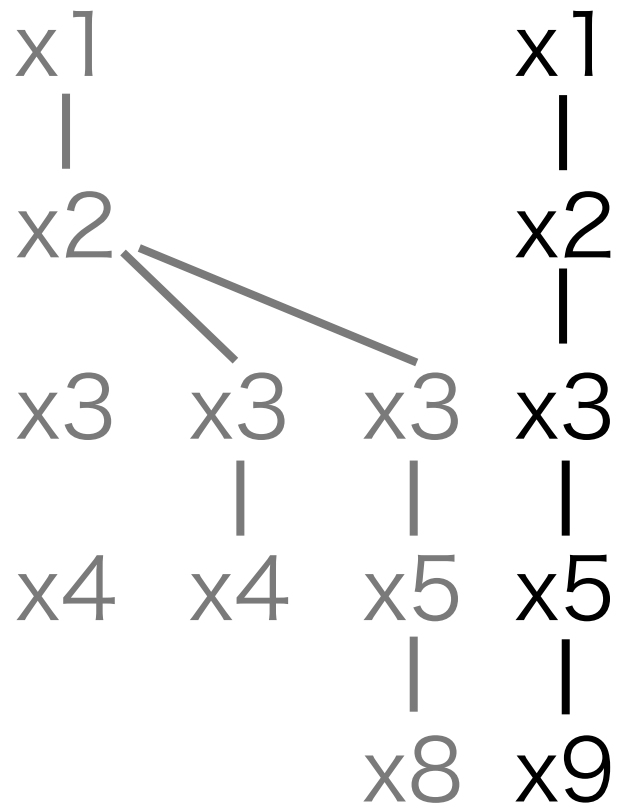
Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Implication Graph

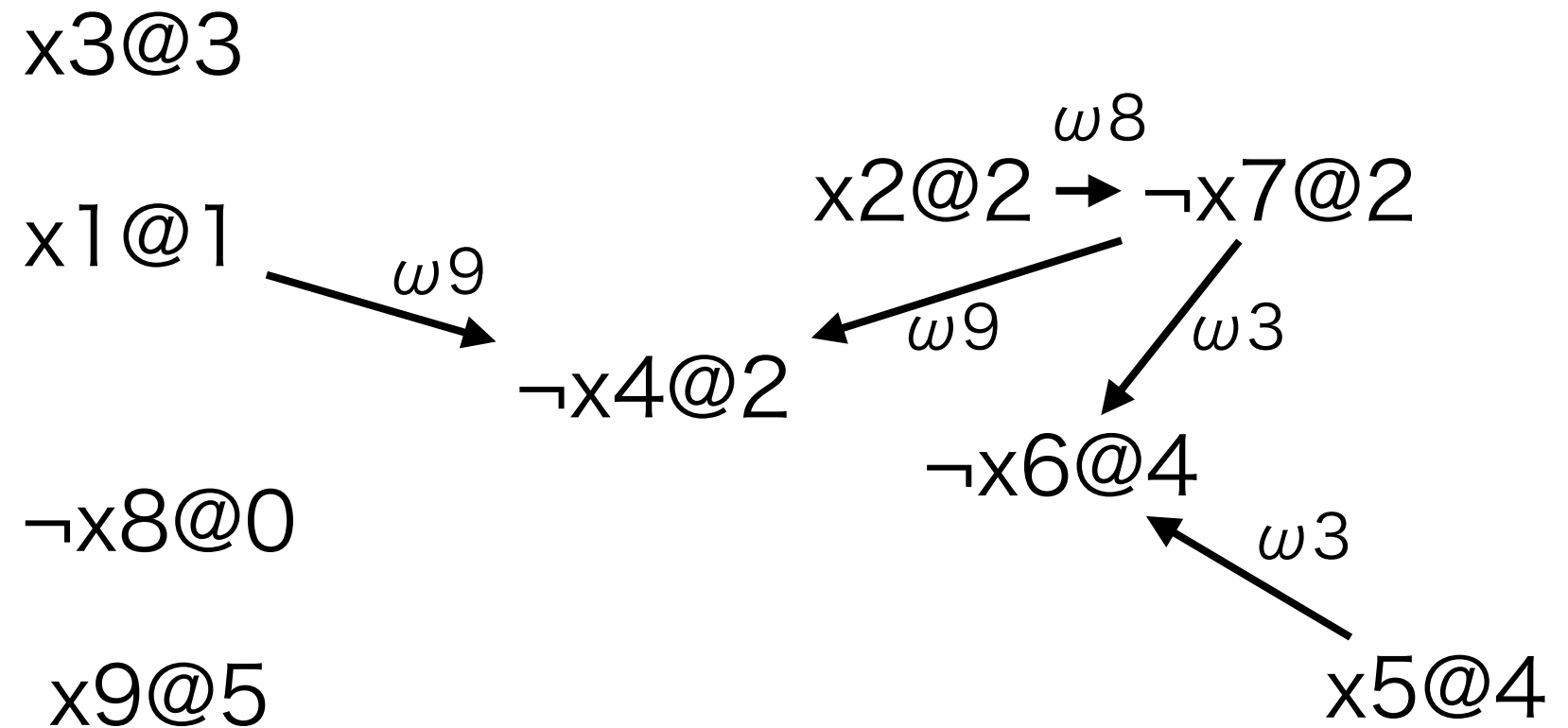
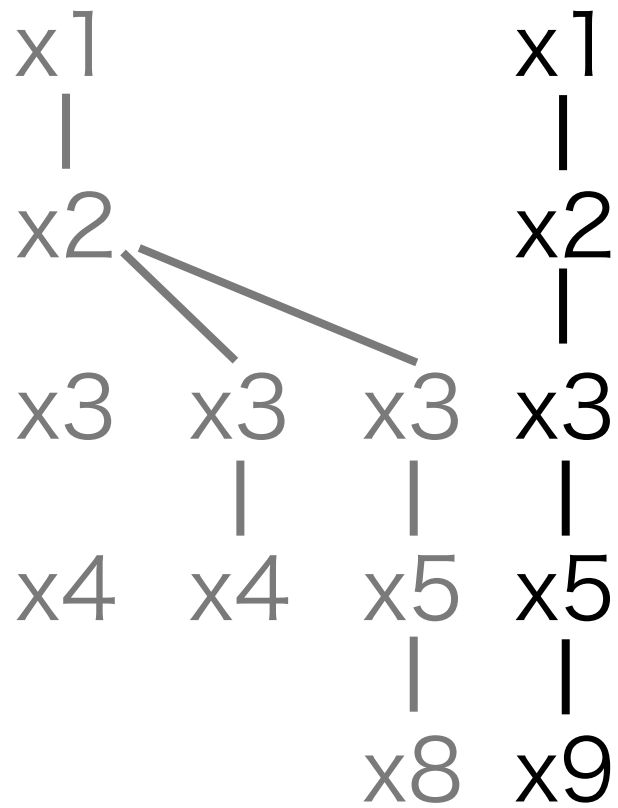


Clause DB

Satisfiable

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Implication Graph



Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

Implication Graph

Found a model

$M =$

$\{ x1, x2, x3, \neg x4, x5, \neg x6, \neg x7, \neg x8, x9 \}$

$x3@3$

$x1@1$

$\neg x8@0$

$x9@5$

$\omega 9$

$\neg x4@2$

$\omega 8$

$x2@2 \rightarrow \neg x7@2$

$\omega 9$

$\omega 3$

$\neg x6@4$

$\omega 3$

$x5@4$

Clause DB

- $\omega 1: \neg x1 \vee \neg x4 \vee x5$
- $\omega 2: \neg x4 \vee x6$
- $\omega 3: \neg x5 \vee \neg x6 \vee x7$
- $\omega 4: \neg x7 \vee x8$
- $\omega 5: \neg x2 \vee \neg x7 \vee x9$
- $\omega 6: \neg x8 \vee \neg x9$
- $\omega 7: \neg x8 \vee x9$
- $\omega 8: \neg x2 \vee \neg x7$
- $\omega 9: \neg x1 \vee \neg x4 \vee x7$
- $\omega 10: \neg x8$

fin.

References

- J. P. Marques-Silva and K. A. Sakallah, "GRASP: a search algorithm for propositional satisfiability," Computers, IEEE Transactions on, vol. 48, no. 5, pp. 506-521, May 1999.
- Handbook of Satisfiability, A. Biere, M. Heule, H. Van Maaren, and T. Walsh, Eds. IOS Press, Feb. 2009.