

p. 56

$$\begin{array}{ccc} A \times_B E & \longrightarrow & E \\ K^*(p) \downarrow \lrcorner & & \downarrow p \\ A & \xrightarrow{K} & B \end{array}$$

$f: X \rightarrow Y$ above $Ku: KI \rightarrow KJ$
 $\text{is } p\text{-cartesian}$
 $\Rightarrow (u, f): (I, X) \rightarrow (J, Y)$ is
 $K^*(p)\text{-cartesian}$

$$(L, Z) \xrightarrow{(u, h)} (I, X) \xrightarrow{(u, f)} (J, Y) \quad \text{with } (u, v, g) \text{ above}$$

$$L \xrightarrow{u} I \xrightarrow{u} J$$

f is cartesian
over Ku

$$Z \xrightarrow{h} X \xrightarrow{f} Y \quad \text{with } g \text{ above}$$

$$KL \xrightarrow{Ku} KI \xrightarrow{Ku} KJ$$

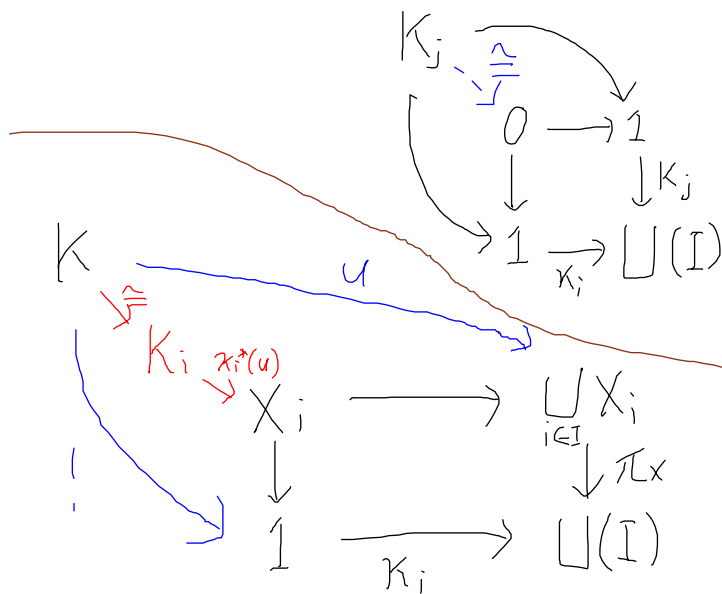
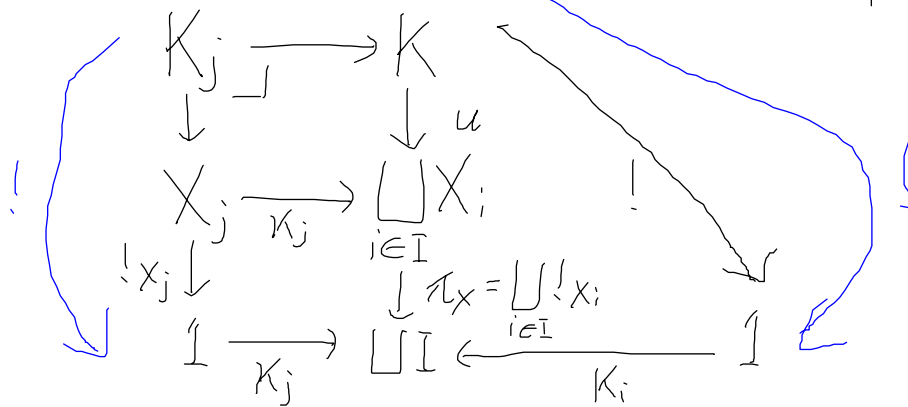
1.5.4 Proposition

$$\begin{array}{ccc} \text{Sets} & \xrightarrow{\sqcup} & \mathbb{C} \\ I & \longmapsto & I \cdot 1 = \bigsqcup_{i \in I} (1) \end{array}$$

$$\text{Sets}(I, \mathbb{C}(1, X)) \cong \mathbb{C}\left(\bigsqcup_{i \in I} (1), X\right)$$

p. 6 | Proposition 1.5-4

$$\pi_X \circ u = \kappa_i \circ !_K$$



for $i \neq j$

よ, 7

$$K \cong \bigcup_{j \in I} K_j \cong K_i$$

p. 62 Lemma 1.5.5 (ii)

$$\begin{array}{c}
 E \\
 \downarrow p \\
 B \\
 \downarrow r \\
 A
 \end{array}
 \quad
 \begin{array}{c}
 E_I \\
 \downarrow p_I \\
 B_I
 \end{array}$$

$$\begin{array}{ccc}
 E_I & \xrightarrow{\quad} & E \\
 p_I \downarrow \lrcorner & & \downarrow p \\
 B_I & \hookrightarrow & B
 \end{array}$$

が "Pullback なので"
 change-of-base
 (=21) p_I は fibration

Exercise 1.5.1

$$\{X_i\}_{i \in I}$$

$\text{FinFam}(\text{Sets})$ なら I が finite

$\text{Fam}(\text{FinSets})$ なら $\forall X_i$ が finite

Exercise 1.5.2

$$\begin{array}{ccc} \mathbb{C} & \xrightarrow{\quad} & \text{UFam}(\omega\text{-Sets}) \\ \downarrow & \lrcorner & \downarrow \\ \text{PER} & \longleftrightarrow & \omega\text{-Sets} \end{array}$$

Exercise 1.5.3

(i) 証明済み

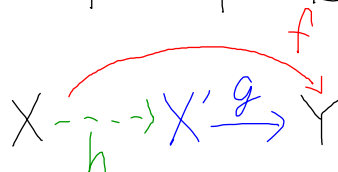
テキストの $I = pX \in \mathcal{B}$ は間違いだよね?
↓

(ii) $f: X \rightarrow Y$ $I = r p X \in \mathcal{A}$

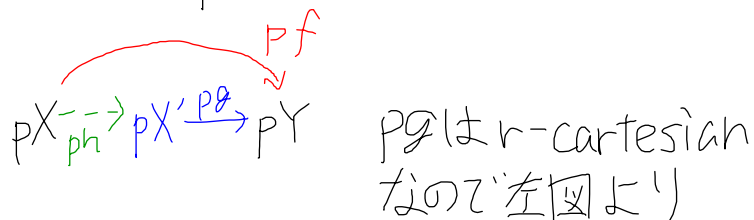


f is p -cartesian $\Rightarrow f = g \circ h$ with
 g is rp -cartesian
 h is p_I -cartesian

Proof
 g is $rp(f)$ rp -cartesian lifting とする



f, g は p -cartesian なので Exercise 1.1.4 (iii) より h は p -cartesian



$$I = I \xrightarrow{rpf} rpY \quad ph \in \mathcal{B}$$

Change-of-base による p_I の定義
より h は p_I -cartesian

$f = g \circ h$ with g rp -cartesian $\Rightarrow f$ is p -cartesian
 h p_I -cartesian

Proof

above iso is $\tilde{S}$ -cartesian

$rp(h) = id_I$ より h は rp -cartesian

cartesian morphism の合成は cartesian
のだから、 $f = g \circ h$ は rp -cartesian

よって f は p -cartesian

Exercise 1.5.4

$\mathbb{C}$ が CCC $\rightarrow S_C(\mathbb{C})$ が CCC

$$\begin{array}{ccc} S_C(\mathbb{C}) & \xrightarrow{\quad} & \mathbf{Sets}^{\rightarrow} \\ \downarrow \text{—} & & \downarrow \text{cod} \\ \mathbb{C} & \xrightarrow{\quad} & \mathbf{Sets} \end{array}$$

Proof

terminal object

$$(1, \lambda x:1. id_1)$$

$\cong 1 \text{ in } \mathbf{Sets}^{\rightarrow}$

Exercise 1.3.7より

$\mathbf{Sets}^{\rightarrow}$ は CCC

products

$$(X, \varphi) \times (Y, \psi) \triangleq (X \times Y, \langle -, - \rangle \circ (\varphi \times \psi))$$

$\cong \varphi \times \psi \text{ in } \mathbf{Sets}^{\rightarrow}$

exponents

$$(X, \varphi) \Rightarrow (Y, \psi) \triangleq (X \Rightarrow Y, \theta : U \rightarrow \mathbb{C}(1, X \Rightarrow Y))$$

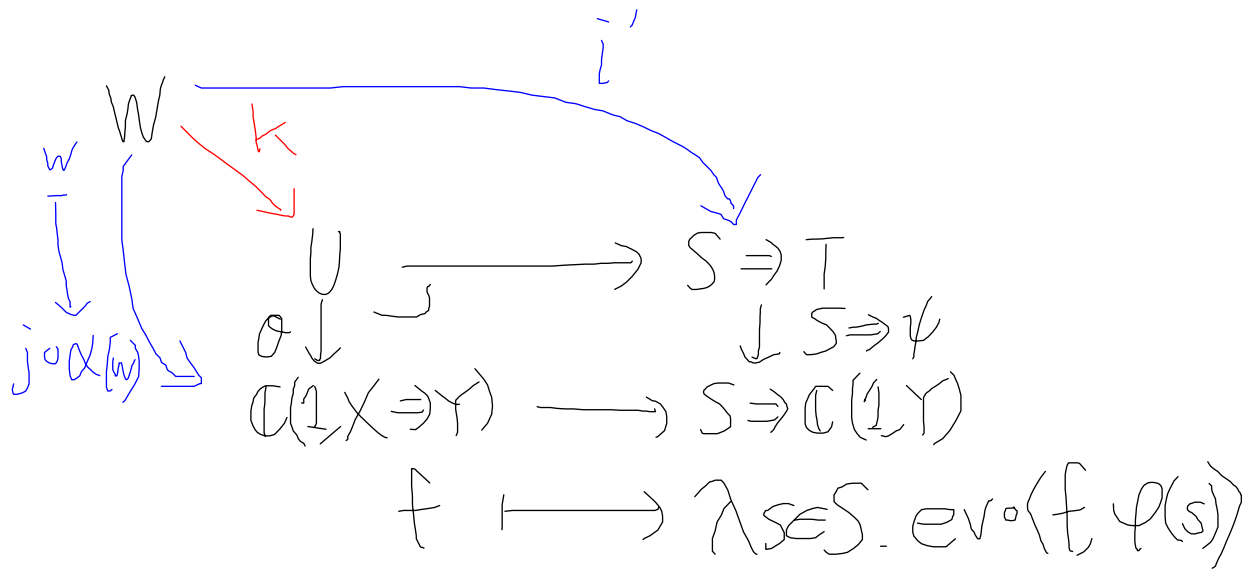
where $S = \text{dom}(\varphi)$, $T = \text{dom}(\psi)$

$$\begin{array}{ccc} U & \xrightarrow{\quad} & S \Rightarrow T \\ \theta \downarrow \text{—} & & \downarrow S \Rightarrow \psi \end{array}$$

$$\mathbb{C}(1, X \Rightarrow Y) \xrightarrow{\quad} S \Rightarrow \mathbb{C}(1, Y)$$

$$f \mapsto \lambda s \in S. \text{ev}_0 \langle f, \varphi(s) \rangle$$

$$\begin{aligned} & S_C(\mathbb{C})((Z, \alpha: W \rightarrow \mathbb{C}(1, Z)) \times (X, \varphi: S \rightarrow \mathbb{C}(1, X)), (Y, \psi: T \rightarrow \mathbb{C}(1, Y))) \\ &= \{(j: Z \times X \rightarrow Y, i: W \times S \rightarrow T) \mid \forall (w, s) \in W \times S. \psi(i(w, s)) = j \circ \langle \alpha(w), \varphi(s) \rangle\} \\ &\cong \{(j': Z \rightarrow (X \Rightarrow Y), i': W \rightarrow (S \Rightarrow T)) \mid \forall w \in W, s \in S. \psi(i'(w)(s)) = \text{ev}_0 \langle j' \circ \alpha(w), \varphi(s) \rangle\} \\ &= \{ \quad \mid \forall w \in W. (S \Rightarrow \psi)(i'(w)) = (\lambda s \in S. \text{ev}_0 \langle j' \circ \alpha(w), \varphi(s) \rangle) \} \\ &\cong \{(j': Z \rightarrow (X \Rightarrow Y), k: W \rightarrow U) \mid \forall w \in W. \theta(k(w)) = j' \circ \alpha(w)\} \\ &= S_C(\mathbb{C})((Z, \alpha), (X, \varphi) \Rightarrow (Y, \psi)) \end{aligned}$$



Exercise 1.5.5

Let A be complete lattice

- coproducts $\bigvee_{i \in I} x_i$ in A is universal $\Rightarrow A$ is a frame

proof

$$\begin{array}{ccc} y \wedge x_i & \longrightarrow & y \wedge \bigvee_{i \in I} x_i \\ \downarrow \lrcorner & & \downarrow \\ x_i & \longrightarrow & \bigvee_{i \in I} x_i \\ & \text{for all } i \in I & \end{array}$$

この $\lrcorner$ universal $\lrcorner$ であるから

$$\bigvee_{i \in I} (y \wedge x_i) = y \wedge \bigvee_{i \in I} x_i$$

よって A は frame

- A is a frame $\Rightarrow$ coproducts $\bigvee_{i \in I} x_i$ are universal

proof

$$\begin{array}{ccc} y_i & \longrightarrow & z \\ \downarrow \lrcorner & & \downarrow \\ x_i & \longrightarrow & \bigvee_{i \in I} x_i \\ & \text{for all } i \in I & \end{array}$$

$y_i = x_i \wedge z$ かつ $z \wedge \bigvee_{i \in I} x_i = z$ より

$$z = z \wedge \bigvee_{i \in I} x_i = \bigvee_{i \in I} (z \wedge x_i) = \bigvee_{i \in I} y_i$$

よって coproducts は universal

Coproducts are disjoint



A has at most 2 elements

とあるが disjoint だと

$$0 \cong X \rightarrow 1$$

$$\downarrow \quad \downarrow$$

$$X \rightarrow X \vee 1$$

となってしまうので

1要素の場合しかダメでは
?

Exercise 1.5.6

$\mathcal{B}$ is distributive category

$\mathcal{C} \nsubseteq \mathcal{B}$:

$$(i) \quad X \times X \xrightarrow{\kappa \times id} (X+Y) \times X$$

∇ split mono

$$(X \times Z) + (Y \times Z) \rightarrow (X+Y) \times Z$$

$[\kappa \times id_Z, \kappa' \times id_Z]$

∇ iso

Proof

$$f = [id, \pi] \circ [\kappa \times id_X, \kappa' \times id_X]^{-1} \in \mathcal{B}$$

$$\begin{aligned} f \circ (\kappa \times id_X) &= [id, \pi'] \circ [\kappa \times id_X, \kappa' \times id_X]^{-1} \circ (\kappa \times id_X) \\ &= [id, \pi'] \circ [\kappa \times id_X, \kappa' \times id_X]^{-1} \circ [\kappa \times id_X, \kappa' \times id_X] \circ \kappa \\ &= [id, \pi'] \circ \kappa \\ &= id \end{aligned}$$

(ii) coprojections $X \xrightarrow{\kappa} X+Y \xleftarrow{\kappa'} Y$ are mono

Proof

$$f, g : Z \rightrightarrows X \text{ with } \kappa \circ f = \kappa \circ g \in \mathcal{B}$$

$$\begin{array}{ccc} \langle f, f \rangle & \xrightarrow{\quad} & X \times X \xrightarrow{\kappa \times id} (X+Y) \times X \\ \downarrow \langle g, f \rangle & & \downarrow \pi \\ Z & \xrightarrow{g} & X \\ f & \searrow & \\ & & X \end{array}$$

$$\kappa \circ f = \kappa \circ g$$

$$\Rightarrow (\kappa \times id) \circ \langle f, f \rangle = (\kappa \times id) \circ \langle g, f \rangle$$

$$\Rightarrow - (\kappa \times id) \notin \text{mono} -$$

$$\langle f, f \rangle = \langle g, f \rangle$$

$$\Rightarrow f = g$$

(iii) $0 \rightarrow 0 \times \mathbb{Z}$ が iso

proof

$$\begin{array}{ccc}
 0 \times \mathbb{Z} & \xrightarrow{\quad id \quad} & 0 \times \mathbb{Z} \\
 \kappa \downarrow & [id, id] & \downarrow \\
 0 \times \mathbb{Z} + 0 \times \mathbb{Z} & \xrightarrow{\quad \quad} & 0 \times \mathbb{Z} \\
 \kappa' \uparrow & & \uparrow \\
 0 \times \mathbb{Z} & \xrightarrow{\quad id \quad} & 0 \times \mathbb{Z}
 \end{array}$$

左図の $[id, id]$ が iso
 なので $\kappa = [id, id]^{-1} = \kappa'$

for any $f, g: 0 \times \mathbb{Z} \Rightarrow Y$

$$\begin{array}{ccc}
 0 \times \mathbb{Z} & \xrightarrow{\quad f \quad} & Y \\
 \kappa \downarrow & [f, g] & \downarrow \\
 0 \times \mathbb{Z} + 0 \times \mathbb{Z} & \xrightarrow{\quad \quad} & Y \\
 \kappa' \uparrow & & \uparrow \\
 0 \times \mathbb{Z} & \xrightarrow{\quad g \quad} & Y
 \end{array}$$

$$\begin{aligned}
 f &= [f, g] \circ \kappa \\
 &= [f, g] \circ \kappa' \\
 &= g
 \end{aligned}$$

よって

$$0 \times \mathbb{Z} \xrightarrow{\pi} 0 \longrightarrow 0 \times \mathbb{Z} = 0 \times \mathbb{Z} \xrightarrow{id} 0 \times \mathbb{Z}$$

一方

$$0 \rightarrow 0 \times \mathbb{Z} \xrightarrow{\pi} 0 = 0 \rightarrow 0 \quad \text{は明らか}$$

ゆえに $0 \times \mathbb{Z} \xrightarrow{\pi} 0$ は $0 \rightarrow 0 \times \mathbb{Z}$ の inverse

(iv) 0 is strict initial object
 すなわち任意の $Z \xrightarrow{f} 0$ は iso

Proof

$$\begin{array}{ccc}
 & f & \rightarrow 0 \\
 Z & \xrightarrow{\langle f, id \rangle} & 0 \times Z \\
 & \downarrow \pi' & \\
 & id & \rightarrow Z
 \end{array}
 \quad \begin{array}{l}
 \cong \uparrow \pi \\
 \parallel_Z
 \end{array}$$

$$\parallel_Z \circ f = \pi' \circ \pi^{-1} \circ f = id_Z$$

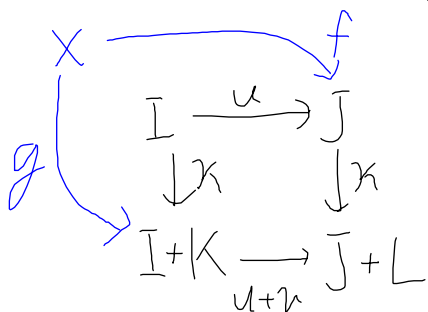
一方、 $f \circ \parallel_Z = id_0$ は明らか

よって f は iso

Exercise 1.5.7

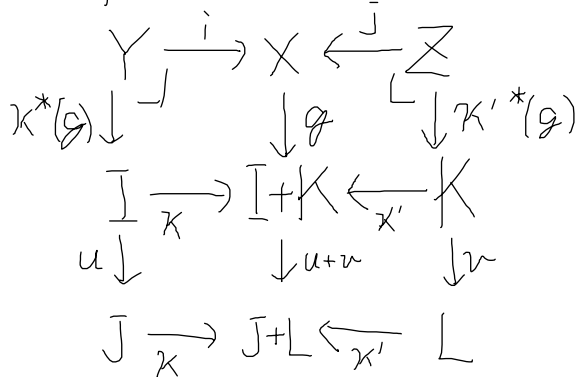
finite limit は仮定して良いんだよね?

Let $\mathcal{B}$ has universal disjoint coproducts

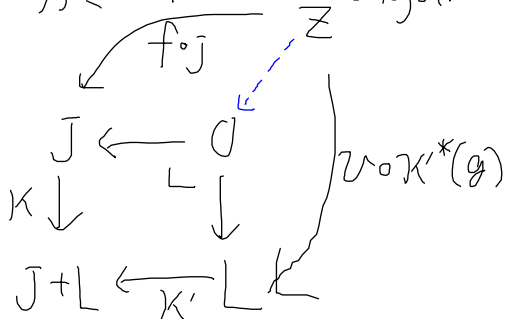


$$\kappa \circ f = (u+v) \circ g$$

coproduct が universal なので以下で定義する $Y \rightarrow X \leftarrow Z$ は coproduct.



一方、coproduct が disjoint なの $Z \rightarrow 0$ が存在



$$\begin{aligned} \kappa \circ f \circ j &= (u+v) \circ g \circ j \\ &= (u+v) \circ \kappa' \circ \kappa^*(g) \\ &= \kappa' \circ v \circ \kappa'(g) \end{aligned}$$

0 が strict initial なの $Z \cong 0$ で $i: Y \xrightarrow{\cong} X$ なの $h: X \rightarrow I$ を $h = \kappa^*(g) \circ i^{-1}$ とおく。 $\kappa \circ h = g$ また $\kappa \circ u \circ h = \kappa \circ u \circ \kappa^*(g) \circ i^{-1}$

$= (u+v) \circ \kappa \circ \kappa^*(g) \circ i^{-1} = (u+v) \circ g = \kappa \circ f$ で κ が mono なの $u \circ h = f$
一意性については X が I が条件を満たすならば $\kappa \circ h' = g = \kappa \circ h$ で κ が mono なの $h' = h$

$$\begin{array}{ccc}
 X & \xrightarrow{\quad} & f \\
 \downarrow g & \searrow u+u & \downarrow \nabla \\
 I+I & \xrightarrow{\quad} & J+J \\
 \downarrow \nabla & & \downarrow \nabla \\
 I & \xrightarrow{u} & J
 \end{array}
 \quad \nabla \circ f = u \circ g$$

$$\begin{array}{ccc}
 Y & \xrightarrow{i} & X \xleftarrow{j} Z \\
 \downarrow \kappa^*(f) & \lrcorner & \downarrow f \\
 J & \xrightarrow{\kappa} & J+J \xleftarrow{\kappa'} J
 \end{array}
 \quad Y+Z \xrightarrow{[i,j]} X$$

$$h = (g \circ i + g \circ j) \circ [i, j]^{-1} : X \rightarrow I+I \quad \text{not } \in \mathcal{C}$$

$$\begin{aligned}
 (u+u) \circ h &= (u+u) \circ (g \circ i + g \circ j) \circ [i, j]^{-1} \\
 &= (u \circ g \circ i + u \circ g \circ j) \circ [i, j]^{-1} \\
 &= (\nabla \circ f \circ i + \nabla \circ f \circ j) \circ [i, j]^{-1} \\
 &= [\kappa \circ \nabla \circ f \circ i, \kappa' \circ \nabla \circ f \circ j] \circ [i, j]^{-1} \\
 &= [f \circ i, f \circ j] \circ [i, j]^{-1} \\
 &= f \circ [i, j] \circ [i, j]^{-1} \\
 &= f
 \end{aligned}$$

$$\begin{aligned}
 \nabla \circ h &= \nabla \circ (g \circ i + g \circ j) \circ [i, j]^{-1} = [g \circ i, g \circ j] \circ [i, j]^{-1} \\
 &= g \circ [i, j] \circ [i, j]^{-1} = g
 \end{aligned}$$

一意性については

$h': X \rightarrow I+I$ が $\nabla \circ h' = g, (u+u) \circ h' = f$ を満たすとする。

$$\begin{array}{ccccc}
 Y & \xrightarrow{i} & X & \xleftarrow{j} & Z \\
 \downarrow h_1 & & \downarrow h' & & \downarrow h_2 \\
 I & \xrightarrow{\kappa} & I+I & \xleftarrow{\kappa'} & I \\
 u \downarrow & \lrcorner & \downarrow u+u & \lrcorner & \downarrow u \\
 J & \xrightarrow{\kappa} & J+J & \xleftarrow{\kappa'} & J
 \end{array}
 \begin{array}{l}
 \left. \begin{array}{c} \\ \\ \\ \end{array} \right\} \kappa^*(f) \quad \left. \begin{array}{c} \\ \\ \\ \end{array} \right\} \kappa'^*(f)
 \end{array}$$

$$\begin{aligned}
 (u+u) \circ h' \circ j &= f \circ j = \kappa \circ \kappa^*(f) \\
 (u+u) \circ h' \circ j &= f \circ j = \kappa' \circ \kappa'^*(f)
 \end{aligned}
 \quad \text{よって} \quad
 \begin{array}{l}
 h'_1: Y \rightarrow I \\
 h'_2: Z \rightarrow I
 \end{array}
 \text{が存在して}$$

$$h' = (h'_1 + h'_2) \circ [i, j]^{-1}$$

ここで

$$\begin{aligned}
 h'_1 &= [h'_1, h'_2] \circ \kappa = \nabla \circ (h'_1 + h'_2) \circ [i, j]^{-1} \circ i \\
 &= \nabla \circ h' \circ i = g \circ i
 \end{aligned}$$

$$\text{同様に } h'_2 = g \circ j$$

$$\text{よって } h' = (g \circ i + g \circ j) \circ [i, j]^{-1} \text{ で一意.}$$

すごく冗長なことをしているけど、面倒なので、このままで

Coproduct functor $+$: $\mathcal{B} \times \mathcal{B} \rightarrow \mathcal{B}$
preserves pullbacks.

proof

$$\begin{array}{ccc} L \xrightarrow{q} K & L' \xrightarrow{q'} K' \\ p \downarrow & \downarrow v \quad p' \downarrow & \downarrow v' \\ I \xrightarrow{u} J & I' \xrightarrow{u'} J' \end{array} \quad \text{とする}$$

$$\begin{array}{ccc} & & \theta \\ & \searrow & \downarrow \\ X & & L+L' \xrightarrow{q+q'} K+K' \\ \downarrow f & & \downarrow p+p' \quad \downarrow v+v' \\ & & I+I' \xrightarrow{u+u'} J+J' \end{array} \quad \begin{array}{l} (u+u') \circ f = (v+v') \circ g \\ \text{が与えられたとする} \end{array}$$

$$\begin{array}{ccc} Y \xrightarrow{i} X \xleftarrow{j} Z \\ \kappa^*(f) \downarrow & \downarrow f & \downarrow \kappa'^*(f) \\ I \xrightarrow{\kappa} I+I' \xleftarrow{\kappa'} I' \end{array}$$

ここで Y, Z, i, j を定義

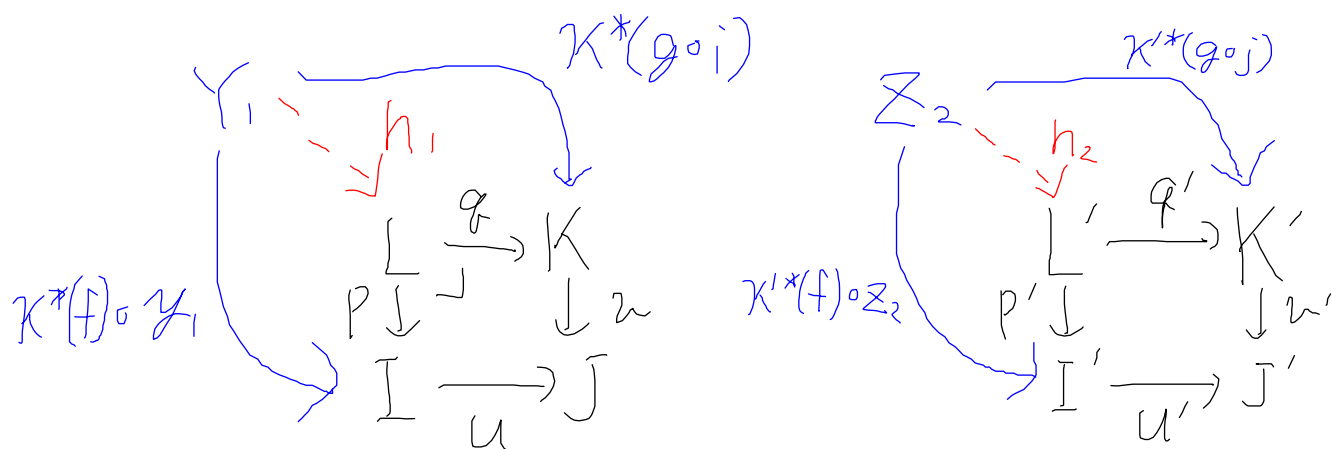
$$f = (\kappa^*(f) + \kappa'^*(f)) \circ [i, j]^{-1}$$

$$\begin{array}{ccc} Y_1 \xrightarrow{\gamma_1} Y \xleftarrow{\gamma_2} Y_2 & & Z_1 \xrightarrow{z_1} Z \xleftarrow{z_2} Z_2 \\ \kappa^*(g_{01}) \downarrow & \downarrow g_{01} & \downarrow \kappa'^*(g_{01}) \quad \kappa^*(g_{02}) \downarrow \quad \downarrow g_{02} \quad \downarrow \kappa'^*(g_{02}) \\ K \xrightarrow{\kappa} K+K' \xleftarrow{\kappa'} K' & & K \xrightarrow{\kappa} K+K' \xleftarrow{\kappa'} K' \quad \text{とすると} \end{array}$$

$$\begin{array}{ccc} Y_2 \xrightarrow{\kappa^*(g_{01})} K' & & \\ \downarrow \gamma_2 & \downarrow \kappa' & \\ Y & \xrightarrow{0} & J' \\ \downarrow \kappa^*(f) & \downarrow & \downarrow \kappa' \\ I \xrightarrow{u} J \xrightarrow{\kappa} J+J' \end{array}$$

$$\begin{aligned} \kappa' \circ \nu \circ \kappa'^*(g_{01}) &= (v+w') \circ \kappa' \circ \kappa'^*(g_{01}) = (v+v') \circ g_{01} \circ \gamma_2 \\ &= (u+u') \circ f \circ i \circ \gamma_2 = (u+u') \circ \kappa \circ \kappa^*(f) \circ \gamma_2 \\ &= \kappa \circ u \circ \kappa^*(f) \circ \gamma_2 \quad \text{よって } \gamma_2 \circ 0, \gamma_1 \circ \gamma_2 \end{aligned}$$

同様にして $Z_1 = 0, Z_2 \cong Z$



$$\begin{aligned} \kappa \circ u \circ \kappa^*(f) \circ \gamma_1 &= (u + u') \circ \kappa \circ \kappa^*(f) \circ \gamma_1 = (u + u') \circ f \circ i \circ \gamma_1 = (v + v') \circ g \circ i \circ \gamma_1 \\ &= (v + v') \circ \kappa \circ \kappa^*(g \circ i) = \kappa \circ v \circ \kappa^*(g \circ i) \end{aligned}$$

$$\kappa \text{ は mono かつ } \gamma \text{ の } \tau \text{ : } u \circ \kappa^*(f) \circ \gamma_1 = v \circ \kappa^*(g \circ i)$$

$$\text{同様にして、 } u' \circ \kappa'^*(f) \circ z_2 = v' \circ \kappa'^*(g \circ j)$$

$$h = X \xrightarrow{[i, j]^{-1}} Y + Z \xrightarrow{\gamma_1^{-1} + z_2^{-1}} Y_1 + Z_2 \xrightarrow{h_1 + h_2} L + L' \text{ とおく}$$

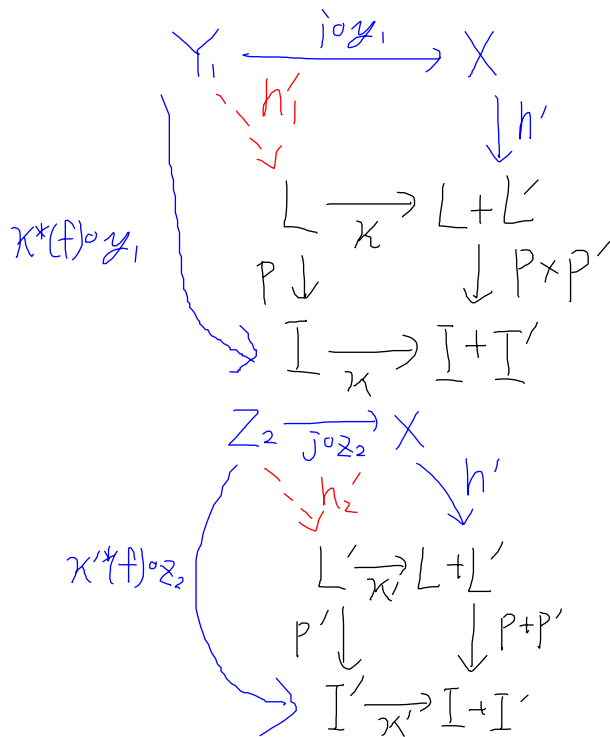
$$\begin{aligned} (p + p') \circ h &= (p \circ h_1 \circ \gamma_1^{-1} + p' \circ h_2 \circ z_2^{-1}) \circ [i, j]^{-1} \\ &= (\kappa^*(f) + \kappa'^*(f)) \circ [i, j]^{-1} = f \end{aligned}$$

$$\begin{aligned} (q + q') \circ h &= (q \circ h_1 \circ \gamma_1^{-1} + q' \circ h_2 \circ z_2^{-1}) \circ [i, j]^{-1} \\ &= [\kappa \circ \kappa^*(g \circ i) \circ \gamma_1^{-1}, \kappa' \circ \kappa'^*(g \circ j) \circ z_2^{-1}] \circ [i, j]^{-1} \\ &= [g \circ i, g \circ j] \circ [i, j]^{-1} = g \circ [i, j] \circ [i, j]^{-1} = g \end{aligned}$$

$\therefore \tau$ h は $L + L'$ は weak pullback

一意性 (= フォー)て

$h' : X \rightarrow L + L'$ が $(p + p') \circ h' = f$
 $(q + q') \circ h' = g$ を満たす
 とする



$$\begin{aligned} \kappa \circ \kappa^*(f) \circ y_1 &= f \circ i \circ y_1 \\ &= (p + p') \circ h' \circ i \circ y_1 \end{aligned}$$

$$\begin{aligned} \kappa' \circ \kappa'^*(f) \circ z_2 &= f \circ j \circ z_2 \\ &= (p + p') \circ h' \circ j \circ z_2 \end{aligned}$$

$$h' = (h_1' + h_2') \circ (y_1^{-1} + z_2^{-1}) \circ [i, j]^{-1}$$

$$\begin{aligned} \kappa \circ p \circ h_1' &= (p + p') \circ \kappa \circ h_1' = (p + p') \circ h' \circ i \circ y_1 = f \circ i \circ y_1 = \kappa \circ \kappa^*(f) \circ y_1 \\ \kappa \text{ が mono なのて } p \circ h_1' &= \kappa^*(f) \circ y_1 \end{aligned}$$

$$\begin{aligned} \kappa \circ q \circ h_1' &= (q + q') \circ \kappa \circ h_1' = (q + q') \circ h' \circ j \circ z_2 = g \circ j \circ z_2 \\ \kappa \text{ が mono なのて } q \circ h_1' &= \kappa^*(g \circ j) = \kappa \circ \kappa^*(g \circ j) \end{aligned}$$

L が pullback なのて $h_1' = h_1$ 同様にして $h_2' = h_2$
 よって $h = h'$

補足: universal $\times$ coproduct があれば
distributive category になる

$$\begin{array}{ccc}
 W & \xrightarrow{g} & Z \times (X+Y) \\
 \downarrow f & \nwarrow \langle \pi \circ g, f \rangle & \downarrow \pi' \\
 Z \times X & \xrightarrow{\kappa} & Z \times (X+Y) \\
 \pi' \downarrow \lrcorner & & \downarrow \pi' \\
 X & \xrightarrow{\kappa} & X+Y
 \end{array}$$

が pullback で $Z \times Y$ も同様

Coproduct が universal $\times$ ので

$$\begin{array}{ccccc}
 Z \times X & \xrightarrow{id \times \kappa} & Z \times (X+Y) & \xleftarrow{id \times \kappa'} & Z \times Y \\
 \pi' \downarrow \lrcorner & & \downarrow \pi' & \lrcorner & \downarrow \pi' \\
 X & \xrightarrow{\kappa} & X+Y & \xleftarrow{\kappa'} & Y
 \end{array}$$

よって、 $Z \times X \xrightarrow{id \times \kappa} Z \times (X+Y) \xleftarrow{id \times \kappa'} Z \times Y$ は
coproduct。

よって $[id \times \kappa, id \times \kappa']$ は iso