p.52

UFam(ω-Sets) は category

$$id_{(X_i,E_i)_{i\in(I,E)}} = (id_{(I,E)}, \{id_{(X_i,E_i)}\}_{i\in I})$$

$\{id_{(X_i,E_i)}\}_{i\in I}$ is $\overbrace{\text{tracked}}^{\text{uniformly}}$ by $\Lambda n.\Lambda m.\ m$

$$\begin{tikzcd}
(X_i,E_i)_{i\in(I,E)} \arrow[r, "{(u,\{f_i\}_{i\in I})}"] \arrow[rr, bend right, "{(v\circ u,\{g_{u(i)}\circ f_i\})}"'] & (Y_j,E_j)_{j\in(J,E)} \arrow[r, "{(v,\{g_j\}_{j\in J})}"] & (Z_k,E)_{k\in(K,E)}
\end{tikzcd}$$

$\left(\begin{array}{l} u \text{ and } v \text{ are tracked by } e_u \text{ and } e_v \\ \{f_i\}_{i\in I} \text{ and } \{g_j\}_{j\in J} \text{ are } \underbrace{\text{tracked by}}_{\text{uniformly}} e_f \text{ and } e_g \end{array}\right.$

$\Rightarrow \left(\begin{array}{l} v\circ u \text{ is tracked by } \Lambda m.\ e_v\cdot(e_u\cdot m) \\ \{g_{u(i)}\circ f_i\}_{i\in I} \text{ is tracked uniformly by} \\ \qquad e = \Lambda n.\Lambda m.(e_g\cdot(e_u\cdot n))\cdot((e_f\cdot n)\cdot m) \end{array}\right.$

$\forall i\in I.\ \forall n\in E(i).\ \forall x\in X_i.\ \forall m\in E(x).$

$\qquad e\cdot n\cdot m \in E_{v(u(i))}(g_{u(i)}(f_i(x)))$

が成り立つのは明らか

p. 54

$$\mathbb{E}(X,Y) \cong \coprod_{u : pX \to pY} \mathbb{E}_{pX}(X, u^*Y)$$

の naturality

$$\begin{tikzcd}
f \in \mathbb{E}(X,Y) \arrow[d, "\mathbb{E}(g{,}h)"'] & \cong & \displaystyle\coprod_{u : pX \to pY} \mathbb{E}_{pX}(X, u^*Y) \ni (pf, f') \arrow[d] \\
h \circ f \circ g \in \mathbb{E}(X',Y') & \cong & \displaystyle\coprod_{v : pX' \to pY'} \mathbb{E}_{pX'}(X', v^*Y')
\end{tikzcd}$$

$$\begin{tikzcd}
X' \arrow[r, "g'"] & X \arrow[d, "f'"'] & & \\
& u^*Y \arrow[r, "\bar{u}"] & Y \arrow[r, "h"] & Y'
\end{tikzcd}$$

$$\begin{tikzcd}
pX' \arrow[r, "pg"'] & pX \arrow[r, "u"] & pY \arrow[r, "ph"'] & pY'
\end{tikzcd}$$

Exercise 1.4.1

$$\begin{tikzcd} \mathbb{E} \arrow[d, "p"'] & \pi^*(X) \arrow[r, "\overline{\pi}"] & X & \delta^*(X) \arrow[r, "\overline{\delta}"] & X \\ \mathbb{B} & I \times J \arrow[r, "\pi"'] & I & I \times J \arrow[r, "\delta"', "=\langle id, \pi' \rangle"' near end] & (I \times J) \times J \end{tikzcd}$$

Family fibration

$$\pi^*(\{X_i\}_{i \in I}) = \{X_i\}_{(i,j) \in I \times J}$$

$$\overline{\pi}(\{X_i\}_{i \in I}) = (\pi, \{id_{X_i}\}_{(i,j) \in I \times J})$$

$$\pi^*(\{f_i\}_{i \in I}) = \{f_i\}_{(i,j) \in I \times J}$$

$$\delta^*(\{X_{i,j,j'}\}_{(i,j,j') \in (I \times J) \times J}) = \{X_{i,j,j}\}_{(i,j) \in I \times J}$$

$$\overline{\delta}(\{X_{i,j,j'}\}_{(i,j,j') \in (I \times J) \times J}) = (\delta, \{id_{X_{i,j,j}}\}_{(i,j) \in I \times J})$$

$$\delta^*(\{f_{i,j,j'}\}_{(i,j,j') \in (I \times J) \times J}) = \{f_{i,j,j}\}_{(i,j) \in I \times J}$$

codomain fibration

$$\begin{tikzcd} \bullet \arrow[r] \arrow[d, "\pi^*(\varphi)"'] \arrow[dr, phantom, "\lrcorner" very near start] & X \arrow[d, "\varphi"] \\ I \times J \arrow[r, "\pi"'] & I \end{tikzcd} \quad \begin{tikzcd} X \arrow[rr, "f"] \arrow[dr, "\varphi"'] & & Y \arrow[dl, "\psi"] \\ & I & \end{tikzcd} \Rightarrow \begin{tikzcd} \bullet \arrow[r] \arrow[d, dashed, "\pi^*(f)"] \arrow[dd, bend right, "\pi^*\varphi"'] & X \arrow[d, "f"] \\ \bullet \arrow[r] \arrow[d, "\pi^*(\psi)"] & Y \arrow[d, "\psi"] \\ I \times J \arrow[r] & I \end{tikzcd}$$

$$\begin{tikzcd} \bullet \arrow[r] \arrow[d, "\delta^*(\varphi)"'] \arrow[dr, phantom, "\lrcorner" very near start] & X \arrow[d, "\varphi"] \\ I \times J \arrow[r, "\delta"'] & (I \times J) \times J \end{tikzcd} \quad \begin{tikzcd} X \arrow[rr, "f"] \arrow[dr, "\varphi"'] & & Y \arrow[dl, "\psi"] \\ & (I \times J) \times J & \end{tikzcd} \quad \begin{tikzcd} \bullet \arrow[r] \arrow[d, dashed, "\delta(f)"] \arrow[dd, bend right, "\delta^*(\varphi)"'] & X \arrow[d, "f"] \\ \bullet \arrow[r] \arrow[d, "\delta^*(\psi)"] & Y \arrow[d, "\psi"] \\ I \times J \arrow[r, "\delta"] & (I \times J) \times J \end{tikzcd}$$

subobject fibrationも同様

Exercise 1.4.1 続き

simple fibrationの場合

$$\pi^*(I, X) = (I \times J, X)$$
$$\overline{\pi}(I, X) = (\pi, \pi')$$
$$\pi^*(id_I, f) = (id_{I \times J}, f \circ (\pi \times id))$$

$$\begin{aligned}
\pi^*(id_{(I,X)}) &= \pi^*(id_I, \pi') \\
&= (id_{I \times J}, \pi' \circ (\pi \times id)) \\
&= (id_{I \times J}, \pi') = id_{\pi^*(I,X)}
\end{aligned}$$

$$\begin{aligned}
\pi^*(id_I, g \circ \langle \pi, f \rangle) &= (id_{I \times J}, g \circ \langle \pi, f \rangle \circ (\pi \times id)) \\
&= (id_{I \times J}, g \circ \langle \pi \circ (\pi \times id), f \circ (\pi \times id) \rangle) \\
&= (id_{I \times J}, g \circ (\pi \times id) \circ \langle \pi, f \circ (\pi, id) \rangle) \\
&= \pi^*(id_I, g) \circ \pi^*(id, f)
\end{aligned}$$

Exercise 1.4.2

Exercise 1.1.3 (ii) と同じ

Exercise 1.4.3

$-^*$ と $-^{*\prime}$ が共に cleavage だとする

$$\begin{tikzcd}
u^{*\prime}(Y) \arrow[r, "\alpha_Y"'] \arrow[rr, bend left, "\bar{u}'(Y)"] & u^*(Y) \arrow[r, "\bar{u}(Y)"'] & Y \\
I \arrow[r, equal] & I \arrow[r] & J
\end{tikzcd}$$

terminal lifting なので $u^*(Y) \cong u^{*\prime}(Y)$

$$\begin{tikzcd}
u^{*\prime}(X) \arrow[r, "\alpha_X"] \arrow[d, "u^{*\prime}(f)"'] & u^*(X) \arrow[r, "\bar{u}(X)"] \arrow[d, "u^*(f)"'] & X \arrow[d, "f"] \\
u^{*\prime}(Y) \arrow[r, "\alpha_Y"'] & u^*(Y) \arrow[r, "\bar{u}(Y)"'] & Y \\
I \arrow[r, equal] & I \arrow[r, "u"'] & J
\end{tikzcd}$$

$u^*(Y) \to Y$ が terminal lifting なので $f \circ \bar{u}(X) \circ \alpha_X$ からの一意な射が存在

$u^*(f) \circ \alpha_X$ は

$\bar{u}(Y) \circ u^*(f) \circ \alpha_X$
$= f \circ \bar{u}(X) \circ \alpha_X$

より条件を満たす

$\alpha_Y \circ u^{*\prime}(f)$ は

$\bar{u}(Y) \circ \alpha_Y \circ u^{*\prime}(f)$
$= \bar{u}'(Y) \circ u^{*\prime}(f)$
$= f \circ \bar{u}'(X) = f \circ \bar{u}(X) \circ \alpha_X$

より条件を満たす。

ゆえに $u^*(f) \circ \alpha_X = \alpha_Y \circ u^{*\prime}(f)$ で

α は natural

Exercise 1.4.4 (if)

$$\begin{tikzcd} U \arrow[r, "h"] \arrow[d, "g"'] \arrow[dr, phantom, "(a)"] & X \arrow[d, "f"] \\ V \arrow[r, "k"'] & Y \end{tikzcd} \quad \text{over} \quad \begin{tikzcd} pU \arrow[r, "ph"] \arrow[d, "pg"'] \arrow[dr, phantom, "(b)"] & pX \arrow[d, "pf"] \\ pV \arrow[r, "pk"'] & pY \end{tikzcd}$$

h and k are cartesian.

(a) is PB ⇐ (b) is PB

$$\begin{tikzcd} W \arrow[drr, bend left, "b"] \arrow[ddr, bend right, "a"'] & & \\ & U \arrow[r, "h"] \arrow[d, "g"'] & X \arrow[d, "f"] \\ & V \arrow[r, "k"'] & Y \end{tikzcd} \qquad \begin{tikzcd} pW \arrow[drr, bend left, "pb"] \arrow[ddr, bend right, "pa"'] \arrow[dr, "u"] & & \\ & pU \arrow[r, "ph"] \arrow[d, "pg"'] \arrow[dr, phantom, "\lrcorner", very near start] & pX \arrow[d, "pf"] \\ & pV \arrow[r, "pk"'] & pY \end{tikzcd}$$

hがcartesianなので

$$\begin{tikzcd} W \arrow[r, "c"'] \arrow[rr, bend left, "b"] & U \arrow[r, "h"'] & X \end{tikzcd}$$

$$\begin{tikzcd} pW \arrow[r, "u"'] & pU \arrow[r, "ph"'] & pX \end{tikzcd}$$

kがcartesianなので

$$\begin{tikzcd} W \arrow[r, "a=g\circ c"'] \arrow[rr, bend left, "f\circ b=k\circ a"] & V \arrow[r, "k"] & Y \end{tikzcd}$$

$$\begin{tikzcd} pW \arrow[r, "\underset{=pg\circ u}{pa}"'] & pV \arrow[r, "pk"'] & pY \end{tikzcd}$$

条件を満たすW→Uは
(b)がPBなので above u で
上の図から一意

Exercise 1.4.4 (only if)

$$\begin{tikzcd} U \arrow[r, "h"] \arrow[d, "g"'] \arrow[dr, phantom, "(a)"] & X \arrow[d, "f"] \\ V \arrow[r, "k"'] & Y \end{tikzcd} \quad \text{over} \quad \begin{tikzcd} pU \arrow[r, "ph"] \arrow[d, "pg"'] \arrow[dr, phantom, "(b)"] & pX \arrow[d, "pf"] \\ pV \arrow[r, "pk"'] & pY \end{tikzcd}$$

(a) is PB $\Rightarrow$ (b) is PB

$$\begin{tikzcd} W \arrow[dr, "pc"] \arrow[drr, bend left, "a"] \arrow[ddr, bend right, "b"'] & & \\ & pU \arrow[r, "ph"] \arrow[d, "pg"'] & pX \arrow[d, "pf"] \\ & pV \arrow[r, "pk"'] & pY \end{tikzcd} \qquad \begin{tikzcd} W' \arrow[dr, "c"] \arrow[drr, bend left, "a'"] \arrow[ddr, bend right, "b'"'] & & \\ & U \arrow[r, "h"] \arrow[d, "g"'] \arrow[dr, phantom, "\lrcorner", very near start] & X \arrow[d, "f"] \\ & V \arrow[r, "k"'] & Y \end{tikzcd}$$

let $W' \xrightarrow{a'} X$ and $W' \xrightarrow{b'} V$ be cartesian lifting of a and b resp.

$$ph \circ pc = p(h \circ c) = pa' = a$$
$$pg \circ pc = p(g \circ c) = pb' = b$$

$W \xrightarrow{d} pU$ が条件を満たすとすると
$W' \xrightarrow{d'} U$ above d が上図を可換にするが
(a)はPBなので $d' = c$ で $d = pc$
よって一意

Exercise 1.4.5 every poset fibration is split

$$\begin{array}{c} \mathbb{E} \\ \downarrow p \\ \mathbb{B} \end{array}
\qquad
\begin{tikzcd}
u^*v^*X \arrow[r] \arrow[d, "\cong"'] & v^*X \arrow[dr, bend left] & \\
(v\circ u)^*X \arrow[rr] & & X
\end{tikzcd}$$

$$I \xrightarrow{u} J \xrightarrow{v} K$$

$\mathbb{E}_I$がposetなので $u^*v^*X = (v\circ u)^*X$

$$\begin{tikzcd}
u^*v^*X \arrow[r, equal] \arrow[d, "u^*v^*f"'] & (v\circ u)^*X \arrow[d, "(v\circ u)^*f"] \\
u^*v^*Y \arrow[r, equal] & (v\circ u^*)Y
\end{tikzcd}$$

$u^*v^*(-)$と$(v\circ u)^*$は自然同型

より、$u^*v^*f = (v\circ u)^*f$

identityに関しても同様

Exercise 1.4.6

$$
\begin{tikzcd}
\mathbb{B} \arrow[r, "K"] & \mathbb{C} & \mathbb{A} \arrow[l, "L"']
\end{tikzcd}
$$

$$
\begin{tikzcd}
(K \downarrow L) \arrow[d, "P"'] \\
\mathbb{B}
\end{tikzcd}
$$

$$
\begin{tikzcd}
KM \arrow[r, "Kv"] \arrow[d, "g"'] & KI \arrow[r, "Ku"] \arrow[d, "f \circ Ku"] & KJ \arrow[d, "f"] \\
LB \arrow[r, "Lh"] \arrow[rr, bend right, "Lh"'] & LA \arrow[r, "Lid"'] & LA \\
M \arrow[r, "v"'] & I \arrow[r, "u"'] & J
\end{tikzcd}
$$

$$u^*(J, A, f) = (I, A, f \circ Ku)$$
$$\bar{u}(J, A, f) = (u, id)$$

$$id^*(J, A, f) = (J, A, f \circ Kid) = (J, A, f)$$

$$
\begin{aligned}
v^*u^*(J, A, f) &= v^*(I, A, f \circ Ku) \\
&= (M, A, f \circ Ku \circ Kv) \\
&= (M, A, f \circ K(u \circ v)) \\
&= (u \circ v)^*(J, A, f)
\end{aligned}
$$

よって split fibration

Exercise 1.4.7

$$\begin{tikzcd} \mathrm{Fam}(\mathbb{C}) \arrow[d] \\ \mathbf{Sets} \end{tikzcd}$$

$$\begin{tikzcd} \{X_{u(j)}\}_{j\in J} \arrow[r, "\{id_{X_{u(j)}}\}_{j\in J}"] \arrow[d, "\{f_{u(j)}\}_{j\in J}"'] & \{X_i\}_{i\in I} \arrow[d, "\{f_i\}_{i\in I}"] \\ \{Y_{u(j)}\}_{j\in J} \arrow[r, "\{id_{Y_{u(j)}}\}_{j\in J}"] & \{Y_i\}_{i\in I} \end{tikzcd}$$

$$J \xrightarrow[u]{} I$$

$\Psi : \mathbf{Sets}^{op} \to \mathbf{Cats}$

$\Psi(I) = \mathbb{C}^I$

$\Psi(u) : \mathbb{C}^I \to \mathbb{C}^J$ for $J \xrightarrow{u} I$ in Sets

$\Psi(u)(\{X_i\}_{i\in I}) = \{X_{u(j)}\}_{j\in J}$

$\Psi(u)(\{f_i\}_{i\in I}) = \{f_{u(j)}\}_{j\in J}$

Exercise 1.4.8

$$\begin{tikzcd}
(\mathbb{B}\downarrow p) = \mathbb{E}\times_{\mathbb{B}}\mathbb{B}^{\to} \arrow[r] \arrow[d, "p^*(\mathrm{cod})"'] & \mathbb{B}^{\to} \arrow[d, "\mathrm{cod}"] \\
\mathbb{E} \arrow[r, "p"'] & \mathbb{B}
\end{tikzcd}$$
in Cats

$$(\mathbb{E}\times_{\mathbb{B}}\mathbb{B}^{\to})((X, u: I\to pX), (Y, v: J\to pY)) \cong \{(f: X\to Y, w: I\to J) \mid v\circ w = pf\circ u\}$$

$$F: \mathbb{E}^{\to}\longrightarrow \mathbb{E}\times_{\mathbb{B}}\mathbb{B}^{\to}$$
$$(f: X'\longrightarrow X)\mapsto (X, pf)$$

$$\begin{tikzcd}
X' \arrow[r, "h"'] \arrow[rr, bend left, "g\circ f"] & u^*(Y) \arrow[r, "\bar{u}(Y)"] & Y
\end{tikzcd}$$

$$\begin{tikzcd}
pX' \arrow[r, "v"'] & I \arrow[r, "u"'] & pY
\end{tikzcd}$$

pをcloven fibrationだとする

$G: \mathbb{E}\times_{\mathbb{B}}\mathbb{B}^{\to}\to\mathbb{E}^{\to}$

$G(X, u: I\to pX) = \bar{u}(X)$ とおくと

$$\begin{aligned}
(\mathbb{E}\times_{\mathbb{B}}\mathbb{B}^{\to})(Ff, (Y, u: I\to pY)) &\cong \{(g: X\to Y,\ v: pX'\to I) \mid u\circ v = pg\circ pf\} \\
&\cong \{(g: X\to Y,\ h: X'\to u^*(Y)) \mid \bar{u}(Y)\circ h = g\circ f\} \\
&\cong \mathbb{E}^{\to}(f, \bar{u}(Y)) = \mathbb{E}^{\to}(f, G(Y, u))
\end{aligned}$$

$$FG(X, u) = F\bar{u}(X) = (X, p\bar{u}(X)) = (X, u)$$

$FG \Rightarrow \mathrm{Id}$ は counit である事は次ページで確認する。

$G: \mathbb{E}\times_{\mathbb{B}}\mathbb{B}^{\to}\to\mathbb{E}^{\to}$ を F の right-inverse right-adjoint とする

$F_X: \mathbb{E}/X\to\mathbb{B}/pX \quad F_X = p$

$G_X: \mathbb{B}/pX\to\mathbb{E}/X \quad G_X = G(X, -)$

とおく

$FG = \mathrm{Id}$ より $\mathrm{cod}(G(X, u)) = X$ かつ $p(G(X, u)) = u$

$\bar{u}(X) : u^*(X) \to X$ を $G_X(u)$ で定義

$$\begin{tikzcd} Y \arrow[r, red, "g"'] \arrow[rr, blue, bend left, "f"] & u^*(X) \arrow[r] & X \\ K \arrow[r, blue, "v"'] & J \arrow[r, "u"'] & I \end{tikzcd}$$

$$\begin{aligned} &\mathbb{B}/pX(pf, u) \\ =\ &\mathbb{B}/pX(F_X f, u) \\ \cong\ &\mathbb{E}/X(f, G_X(u)) \\ =\ &\mathbb{E}/X(f, \bar{u}(X)) \end{aligned}$$

$$\begin{tikzcd} K \arrow[rr, "v"] \arrow[dr, "pf"'] & & J \arrow[dl, "u"] \\ & I & \end{tikzcd} \text{ in } \mathbb{B}/pX \iff \begin{tikzcd} Y \arrow[rr, "g"] \arrow[dr, "f"'] & & u^*(X) \arrow[dl, "\bar{u}(X)"] \\ & X & \end{tikzcd} \text{ in } \mathbb{E}/X$$

$v = \varepsilon_f \circ F_X(g) = pg$ なので g は above v

$\therefore$ G は cleavage

$FG \Rightarrow Id$ が counit であることの確認

$$\begin{tikzcd} u^*(Y) \arrow[r, blue, "id"] \arrow[d, "\bar{u}(Y)"'] & u^*(Y) \arrow[d, "\bar{u}(Y)"] \\ Y \arrow[r, "id"'] & Y \end{tikzcd} \text{ in } \mathbb{E}^{\to} \iff \begin{tikzcd} p\,u^*(Y) \arrow[r, "id"] \arrow[d, "p\bar{u}(Y)"'] & I \arrow[d, "u"] \\ pY \arrow[r, "pid"] & pY \end{tikzcd} \text{ in } \mathbb{E} \times_{\mathbb{B}} \mathbb{B}^{\to}$$

$$Y \xrightarrow{id} Y$$

$id_{G(Y,u)} \in \mathbb{E}^{\to}(G(Y,u), G(Y,u))$

$\varepsilon_{(Y,u)} \in (\mathbb{E} \times_{\mathbb{B}} \mathbb{B}^{\to})(FG(Y,u), (Y,u))$

Exercise 1.4.9

$$\begin{tikzcd} X \arrow[r, bend left, "id"] \arrow[d, "\eta_I"'] & X \\ id^*(X) \arrow[ur, "\overline{id}(X)"'] & \end{tikzcd} \qquad \begin{tikzcd} u^*v^*(X) \arrow[r] \arrow[d, "\mu_{u,v}"'] & v^*(X) \arrow[d] \\ (v\circ u)^*(X) \arrow[r] & X \end{tikzcd}$$

$$I \xrightarrow{id} I \qquad I \xrightarrow{u} J \xrightarrow{v} K$$

for $u : I \to J$

$$\begin{tikzcd} & u^* \arrow[dl, "\eta_I u^*"'] \arrow[d, equal] \arrow[dr, "u^*\eta_J"] & \\ (id_I)^*u^* \arrow[r, "\mu_{id_I,u}"'] & u^* & u^*(id_J)^* \arrow[l, "\mu_{u,id_J}"] \end{tikzcd}$$

$$\begin{tikzcd} u^*(X) \arrow[d, "\eta_I u^* X"'] \arrow[dr, bend left, "id"] \arrow[dd, bend right=60, "id"'] & & \\ (id_I)^*(u^*(X)) \arrow[r] \arrow[d, "\mu_{id_I,u} X"'] & u^*(X) \arrow[dr, "\overline{u}(X)"] & \\ u^*(X) \arrow[rr] & & X \end{tikzcd}$$

より $\mu_{id_I,u} \circ \eta_I u^* = id$

同様に $\mu_{u,id_I} \circ u^*\eta_J = id$

$$\begin{tikzcd} u^*v^*w^* \arrow[r, "u^*\mu_{v,w}"] \arrow[d, "\mu_{u,v}w^*"'] & u^*(w\circ v)^* \arrow[d, "\mu_{u,w\circ v}"] \\ (v\circ u)^*w^* \arrow[r, "\mu_{v\circ u,w}"'] & (w\circ v\circ u)^* \end{tikzcd}$$

for $I \xrightarrow{u} J \xrightarrow{v} K \xrightarrow{w} L$

$$\begin{tikzcd} u^*v^*w^*X \arrow[r] \arrow[d] & v^*w^*X \arrow[r] & w^*X \arrow[d] \\ (w\circ v\circ u) \arrow[rr] & & X \end{tikzcd}$$

$$I \xrightarrow{u} J \xrightarrow{v} K \xrightarrow{w} L$$

$\mu_{v\circ u,w} \circ \mu_{u,v}w^*$ と $\mu_{u,w\circ v} \circ u^*\mu_{v,w}$ は共に上図を可換にするので等しい。

Exercise 1.4.10

Indexed category $\Psi : \mathbb{B}^{op} \to Cat$

Category $\int \Psi$

object (I, X) where $I \in |\mathbb{B}|$, $X \in |\Psi(I)|$

morphism $(u, f) : (I, X) \to (J, Y)$

where $u \in \mathbb{B}(I, J)$

$f \in \Psi(I)(X, \Psi(u)(Y))$

composition $(I, X) \xrightarrow{(u,f)} (J, Y) \xrightarrow{(v,g)} (K, Z)$

$= (v \circ u, \Psi(u)(g) \circ f)$

$p : \int \Psi \to \mathbb{B}$

$p(I, X) = I$

$p(u, f) = u$

$$
\begin{tikzcd}
(K,Z) \arrow[r, "{(v,f)}"'] \arrow[rr, bend left, "{(u \circ v, f)}"] & \underset{u^*(J,Y) \,=}{(I, \Psi(u)(Y))} \arrow[r, "{\bar{u}(Y) \,=\, (u,\, id_{\Psi(u)(Y)})}"] & (J, Y)
\end{tikzcd}
$$

$f \in \Psi(K)(Z, \Psi(u \circ v)Y)$

$= \Psi(Z, \Psi(v)\Psi(u)(Y))$

$$K \xrightarrow{v} I \xrightarrow{u} J$$

よって p は cloven fibration

$id^*(J, Y) = (J, \Psi(id)(Y)) = (J, Y)$

$(u \circ v)^*(J, Y) = (J, \Psi(u \circ v)(Y)) = (J, \Psi(v)\Psi(u)(Y)) = v^* u^*(J, Y)$

よって p は split